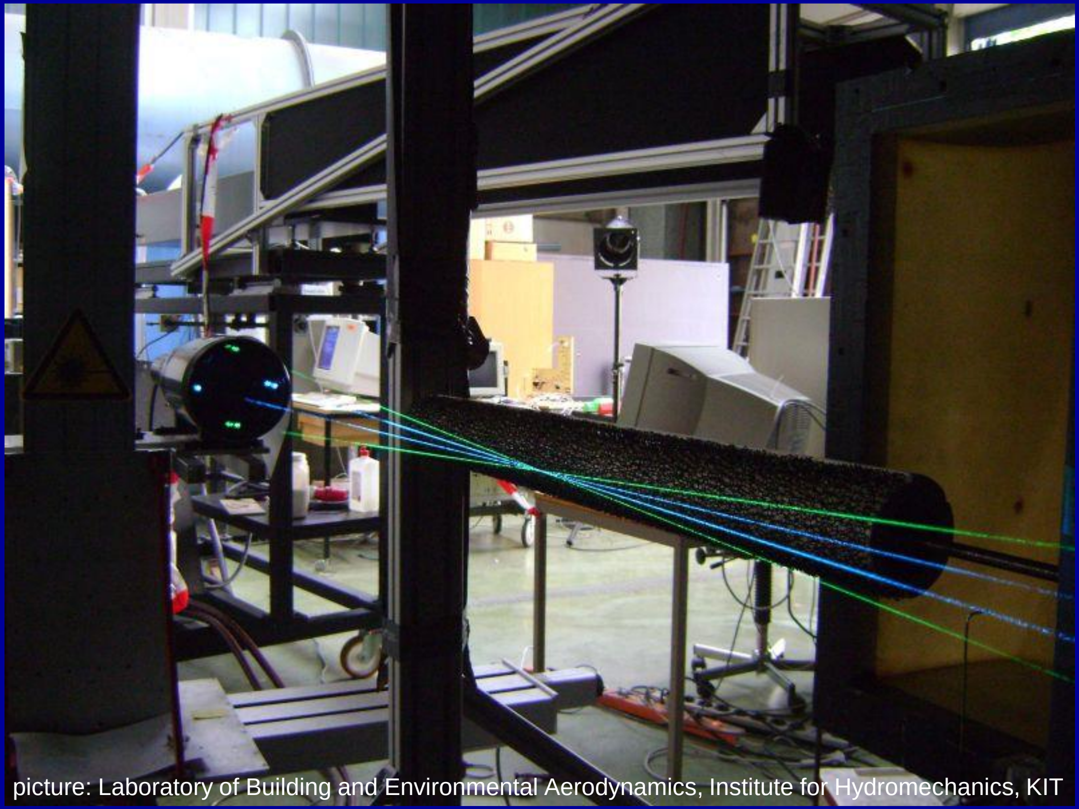
Laser Doppler Velocimetry - LDV

(Laser Doppler Anemometry - LDA)



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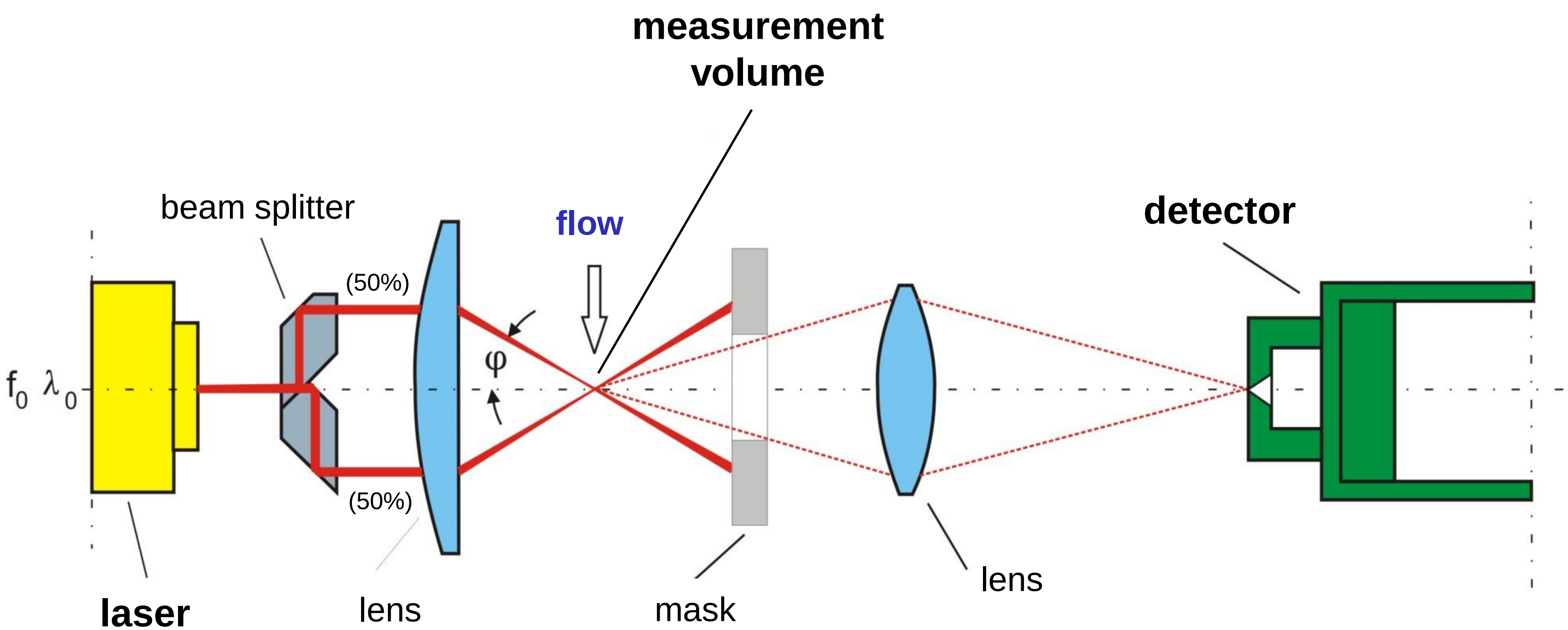
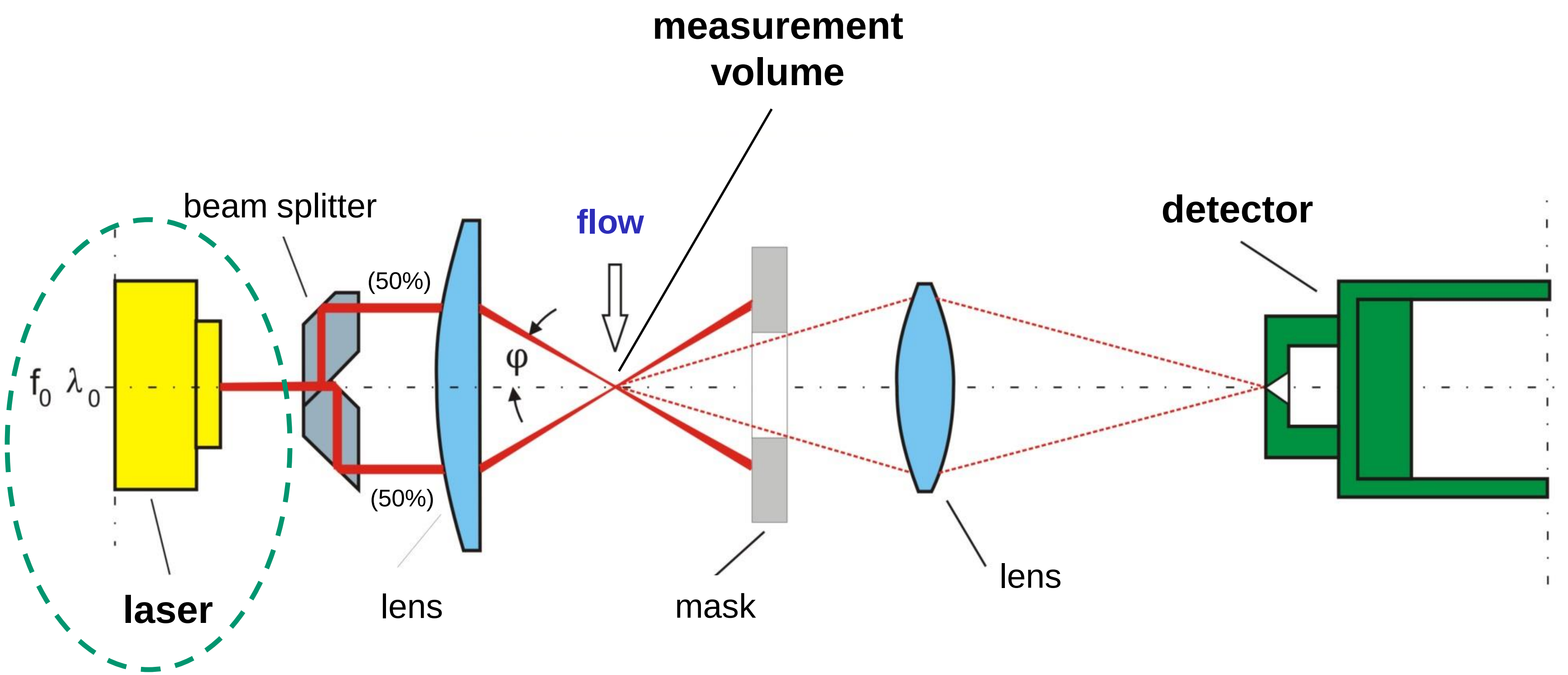
Schematic setup of LDV system

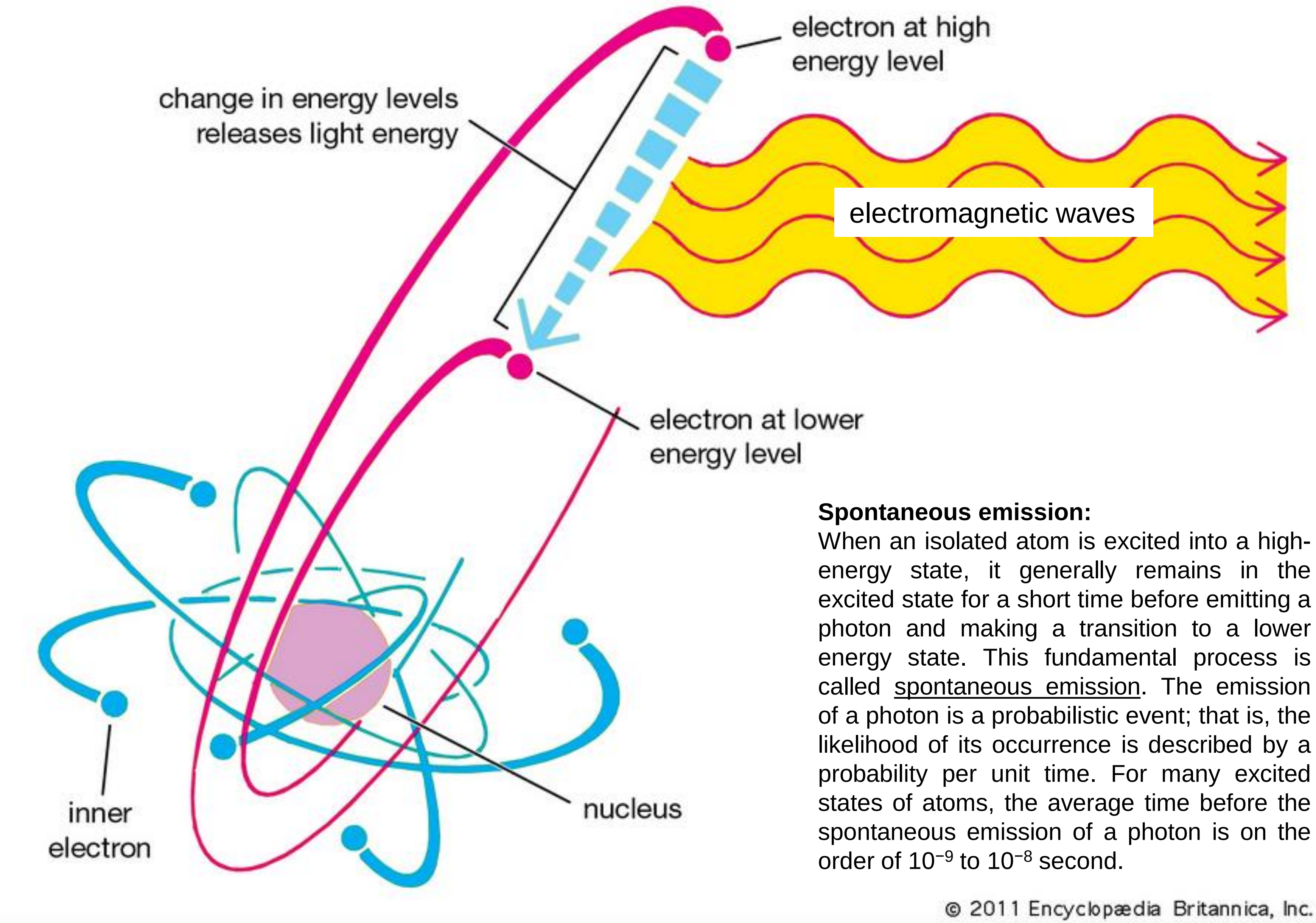


Laser

Schematic setup of LDV system



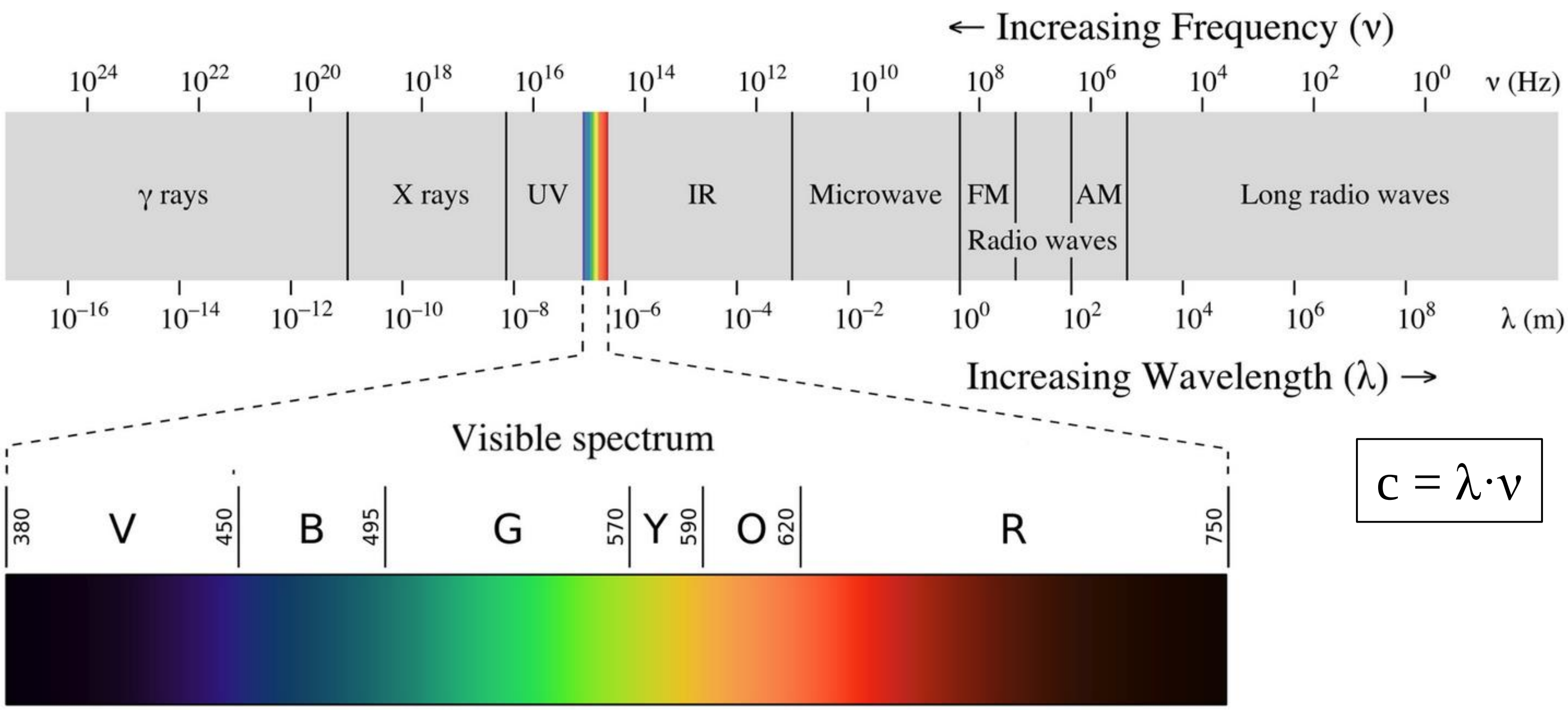
Light / Electromagnetic Waves



Spontaneous emission:
When an isolated atom is excited into a high-energy state, it generally remains in the excited state for a short time before emitting a photon and making a transition to a lower energy state. This fundamental process is called <u>spontaneous emission</u>. The emission of a photon is a probabilistic event; that is, the likelihood of its occurrence is described by a probability per unit time. For many excited states of atoms, the average time before the spontaneous emission of a photon is on the order of 10^{-9} to 10^{-8} second.

Electromagnetic Spectrum and Visible Light

Light is the visible part of the electromagnetic spectrum.



$$c = \lambda \cdot \nu$$

https://commons.wikimedia.org/wiki/File:EM_spectrumrevised.png#/media/File:EM_spectrumrevised.png

LASER = Light Amplification by Stimulated Emission of Radiation

photoelectric effect:	A. Einstein (1905)
first laser effects in laboratory:	T. Maimann (1960), Hughes Aircraft Company
first gas laser:	A. Javan et al. (1961), Bell Telephone Lab

Lasers are light sources with special features. Their radiation is:

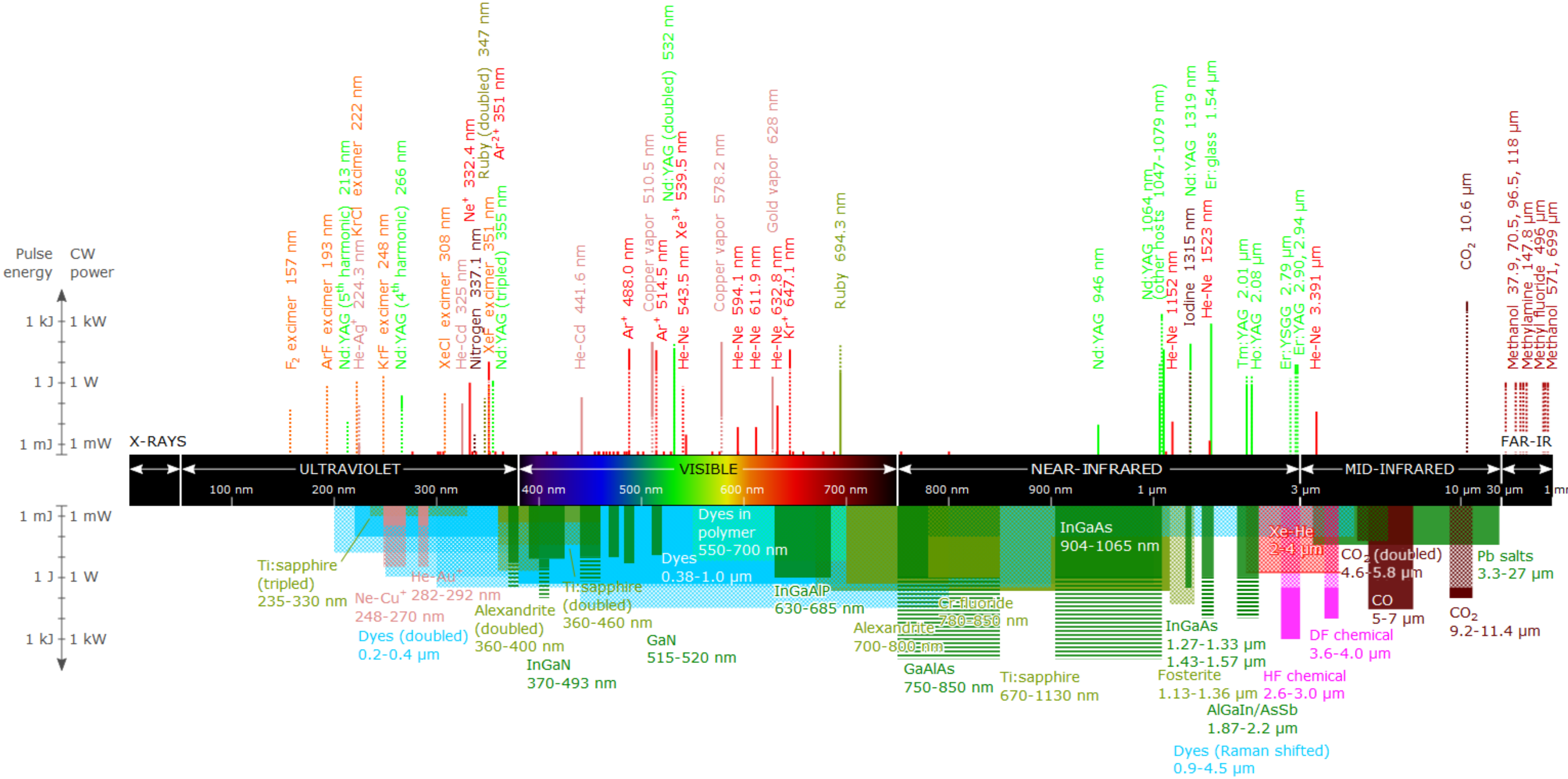
- emitted by stimulation (→ stimulated emission)
- monochromatic
- coherent
- collimated (directed, low divergence)
- usually linearly polarized (E-field oscillates in fixed plane)
- high energy / intensity

Radiation emitted by lasers can be distinguished according to:

- radiation in the visible spectrum (400 – 750 nm)
- radiation in the non-visible spectrum
- continuous laser / wave (cw) → LDV
- pulsed laser / wave → PIV

Laser Types

gas lasers, chemical lasers, dye lasers, metal-vapor lasers, solid-state lasers, semiconductor laser, other types of lasers

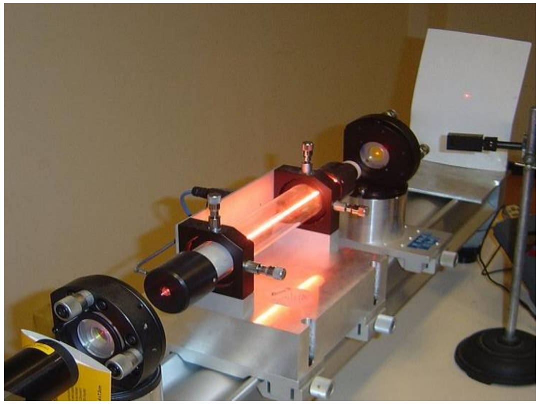


https://commons.wikimedia.org/wiki/File:Commercial_laser_lines.svg#/media/File:Commercial_laser_lines.svg

Laser types (commonly) deployed in Fluid Mechanics

- He-Ne (Helium-Neon, gas)
- Ar^+ (Argon-Ion, gas)
- Nd:YAG (neodymium:yttrium aluminum garnet, solid-state)

Laser Setup

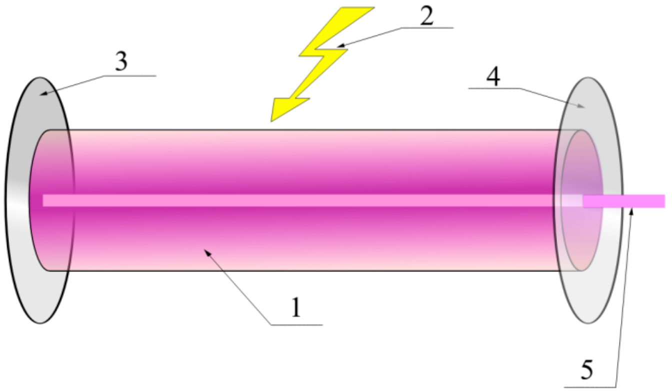


https://en.wikipedia.org/wiki/Laser#/media/
File:Laser_DSC09088.JPG

active medium also called:

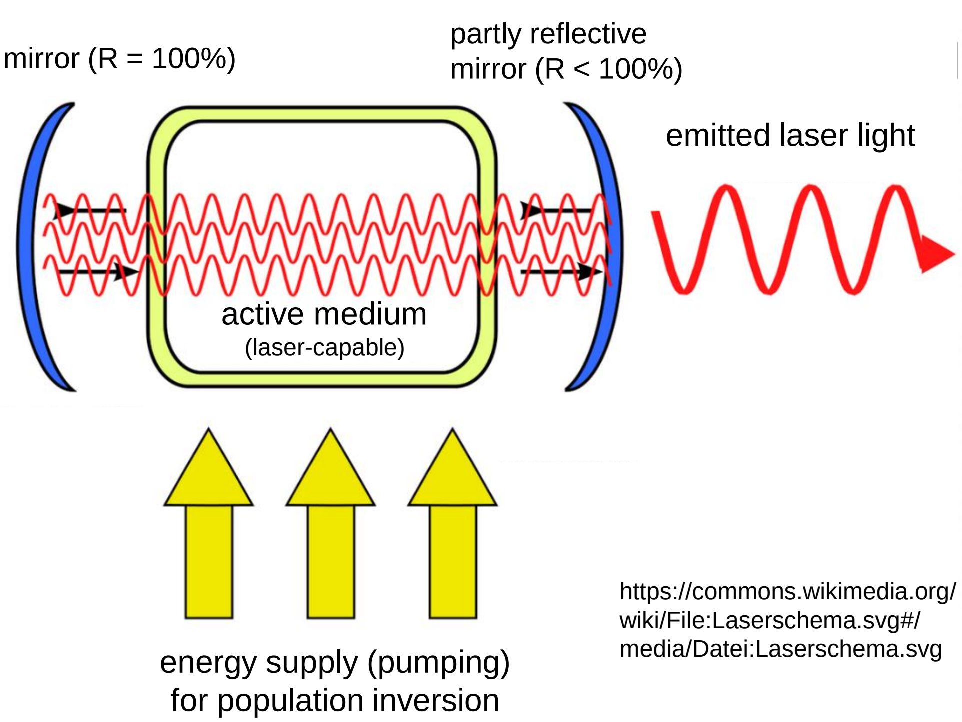
- gain medium
- lasing medium

resonator (optical cavity) setup



1 gain medium
2 pumping energy
3 high reflector
4 output coupler
5 Laser beam

https://en.wikipedia.org/wiki/Laser#/media/File:Laser.svg



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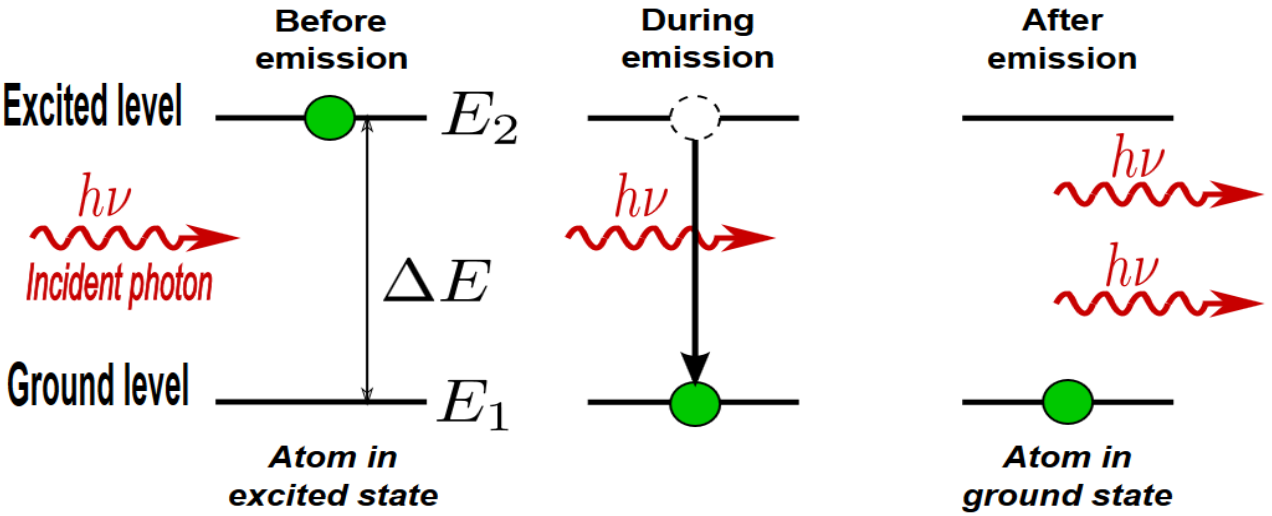
Stimulated Emission (vs. spontaneous emission)



$$E_2 - E_1 = \Delta E = h\nu$$

https://commons.wikimedia.org/w/index.php?curid=3983414

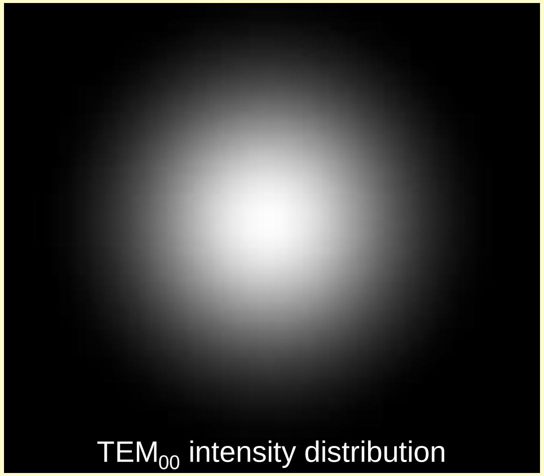
> h = 6.6260×10^{-34} J·s
> (Planck constant)
>
> v: frequency of radiation

- electrons 'prefer' lower energetic level (ground level)
- electrons have to be 'lifted' (pumped) on a higher energetic level

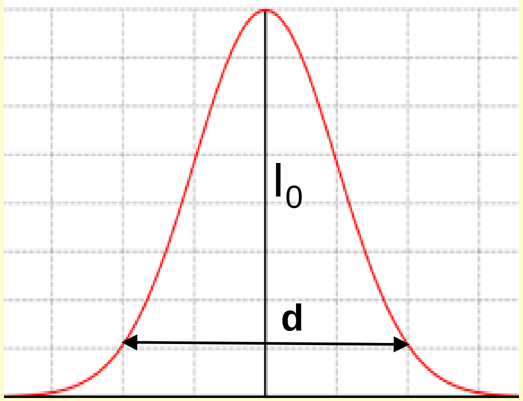
  → energy input (aka pumping) is required
- **population inversion**: #atoms in excited state n_2 > #atoms in ground state n_1
- in order to achieve population inversion, 3 or 4 level energy systems are required

Characteristics of Laser Beams – Intensity Distribution and TEM

Intensity distribution in cross section

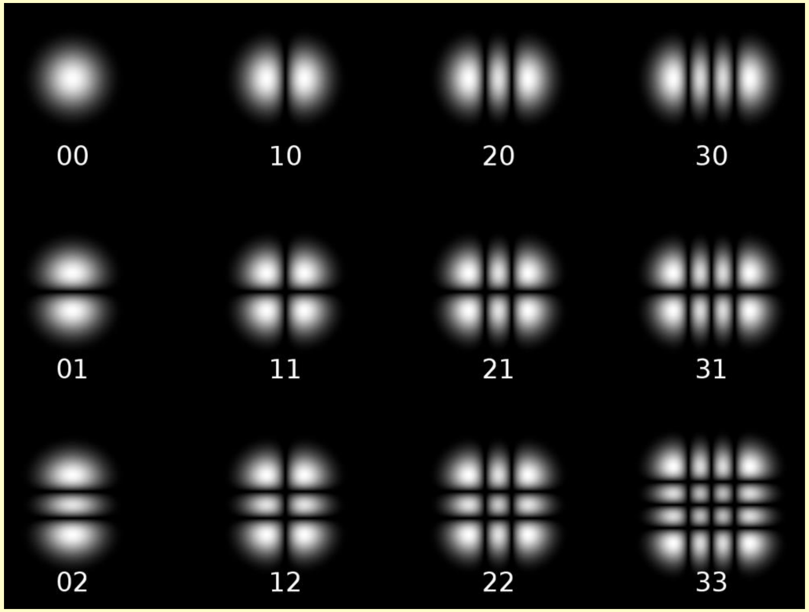


TEM_{00} intensity distribution



d: nominal beam diameter
where $I_{lateral} = 1/e^2 \, I_0$

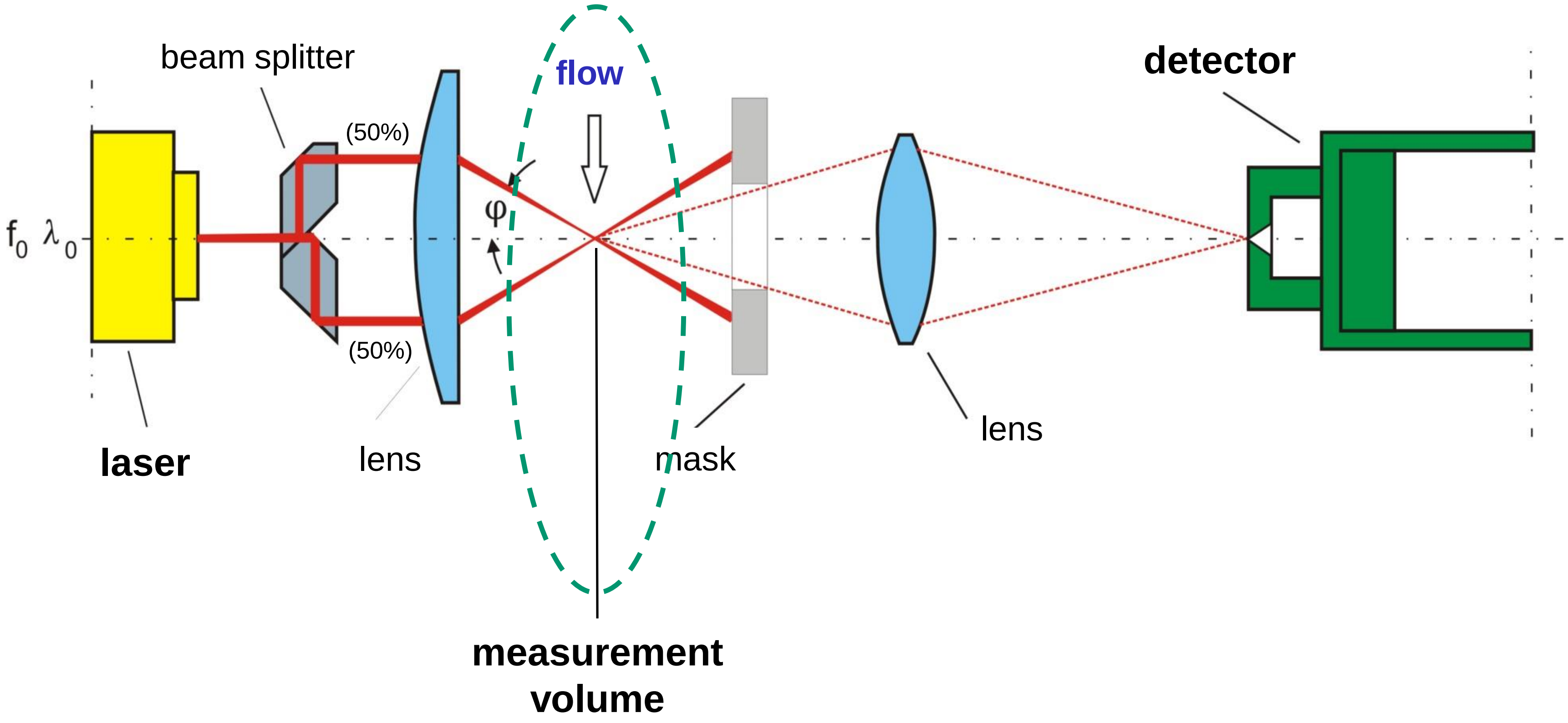
Transverse electromagnetic modes (TEM)



Transverse electromagnetic modes (TEM) according to different resonance conditions. They depend on the design of the resonator (on the shape of the mirrors).

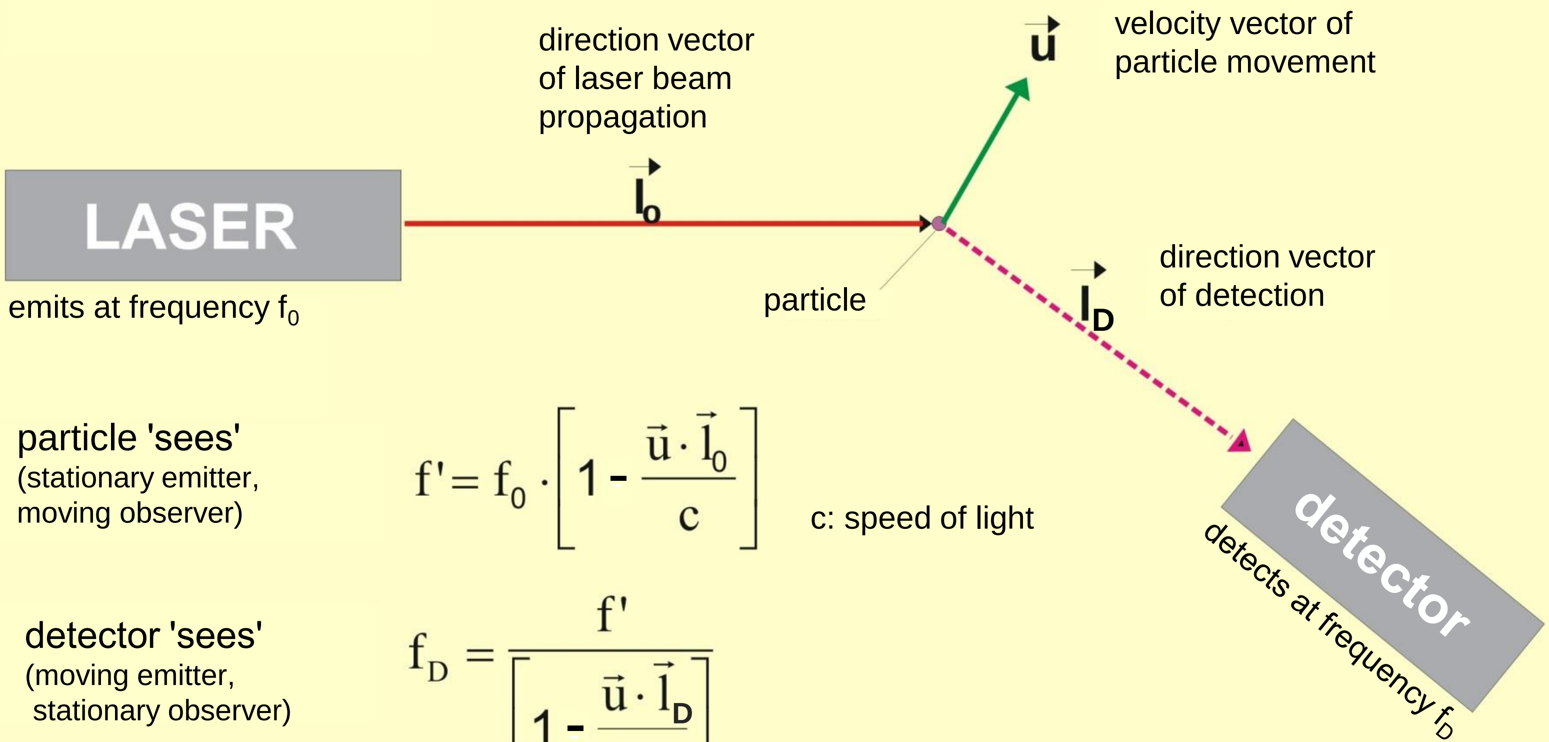
Measurement Volume

Schematic setup of LDV system



Doppler Effect(s)

(Christian Doppler 1842)



particle 'sees'
(stationary emitter,
moving observer)

$$f' = f_0 \cdot \left[1 - \frac{\vec{u} \cdot \vec{l}_0}{c}\right]$$

c: speed of light

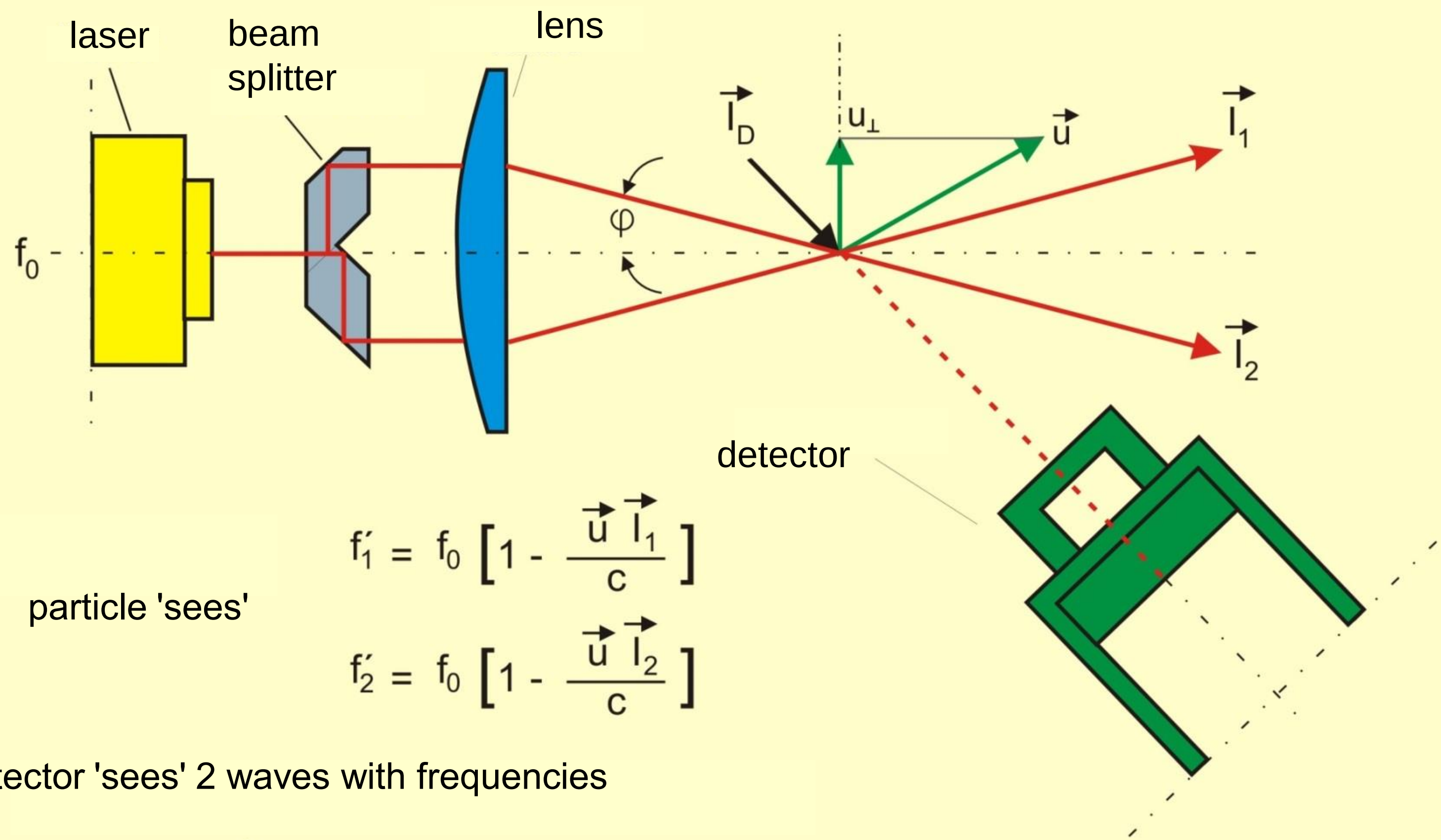
detector 'sees'
(moving emitter,
stationary observer)

$$f_D = \frac{f'}{\left[1 - \frac{\vec{u} \cdot \vec{l}_D}{c}\right]}$$

Taylor series expansion

$$f_D = f_0 \cdot \left[1 - \frac{\vec{u} \cdot \vec{l}_0}{c} + \frac{\vec{u} \cdot \vec{l}_D}{c}\right]$$

Laser Doppler Velocimetry Setup



particle 'sees'

$$f_1' = f_0 \left[1 - \frac{\vec{u}\,\vec{l}_1}{c}\right]$$

$$f_2' = f_0 \left[1 - \frac{\vec{u}\,\vec{l}_2}{c}\right]$$

detector 'sees' 2 waves with frequencies

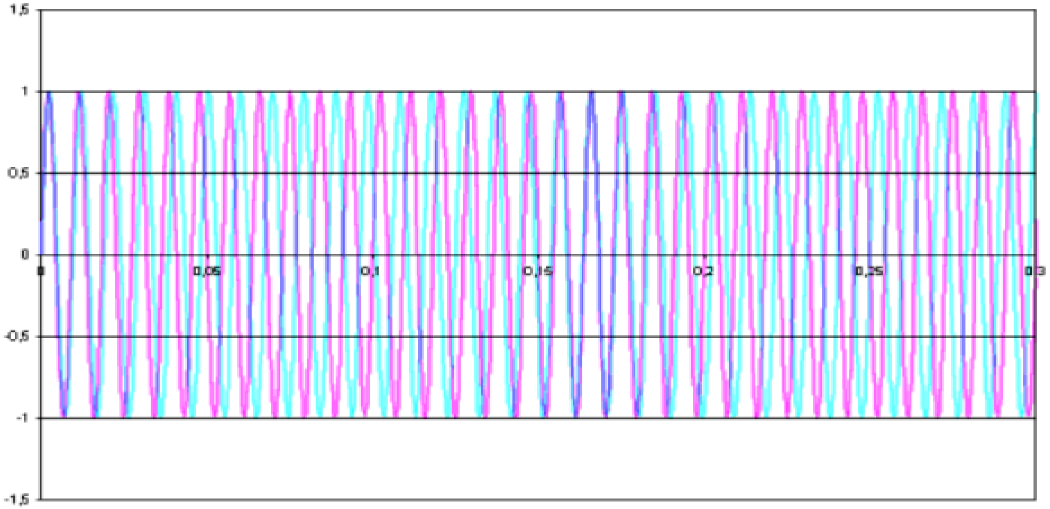
$$f_{D_1} = \frac{f_1'}{1 - \frac{\vec{u}\,\vec{l}_D}{c}}$$

$$f_{D_2} = \frac{f_2'}{1 - \frac{\vec{u}\,\vec{l}_D}{c}}$$

detector 'sees' superposition of 2 waves with <u>beat frequency</u> (follows from Taylor series expansion)

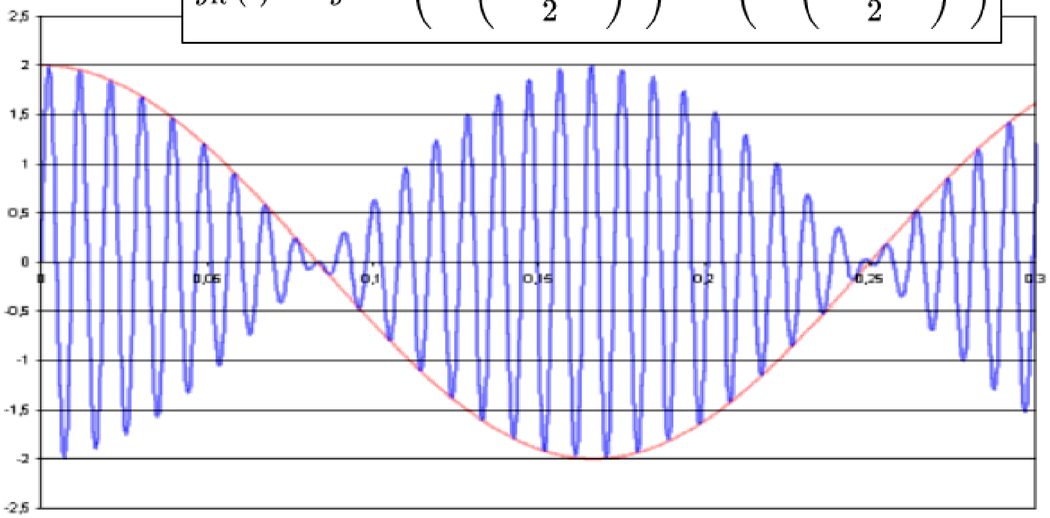
$$\Delta f = f_{D_1} - f_{D_2} \approx f_0 \left[\frac{\vec{u}\,\vec{l}_2 - \vec{u}\,\vec{l}_1}{c}\right] = f_0 K u_\perp$$

Beat Frequency



two sinusoidal waves of slightly different frequencies f_1 and f_2
(same amplitude)

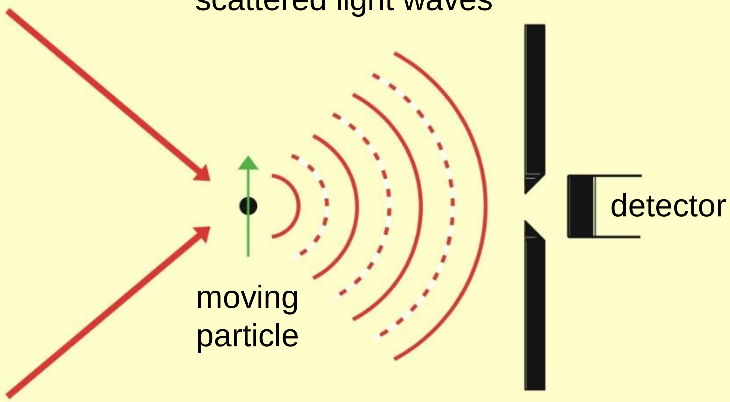
$$y_{\mathrm{R}}(t) = 2\hat{y} \cdot \sin\left(2\pi\left(\frac{f_1 + f_2}{2}\right)t\right) \cdot \cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)$$



https://en.wikipedia.org/wiki/Beat_(acoustics)#/media/File:Beat.png

envelope (red line) is
beat frequency Δf
(*$\Delta f = f_D$: Doppler frequency*)

Explanatory Modell I: Scalar Wave Theory of Light



superposition of light waves scattered from moving particle

→ signal with 'Doppler frequency'
- signal contains velocity information

$$E_1 = E_0 \cdot \cos\ 2\pi \left(f_1 \cdot t - \frac{x}{\lambda_1} \right)$$

$$E_2 = E_0 \cdot \cos\ 2\pi \left(f_2 \cdot t - \frac{x}{\lambda_2} \right)$$

(E: electric field)

Superposition of scattered light waves:

$$E_{1/2} = 2 \cdot E_0 \cdot \cos\ 2\pi \left(\frac{f_1 + f_2}{2} \cdot t - \frac{\lambda_1 + \lambda_2}{2 \cdot \lambda_1 \cdot \lambda_2} \cdot x \right) \cdot$$

$$\cos\ 2\pi \left(\frac{f_1 - f_2}{2} \cdot t - \frac{\lambda_1 - \lambda_2}{2 \cdot \lambda_1 \cdot \lambda_2} \cdot x \right)$$

beat frequency

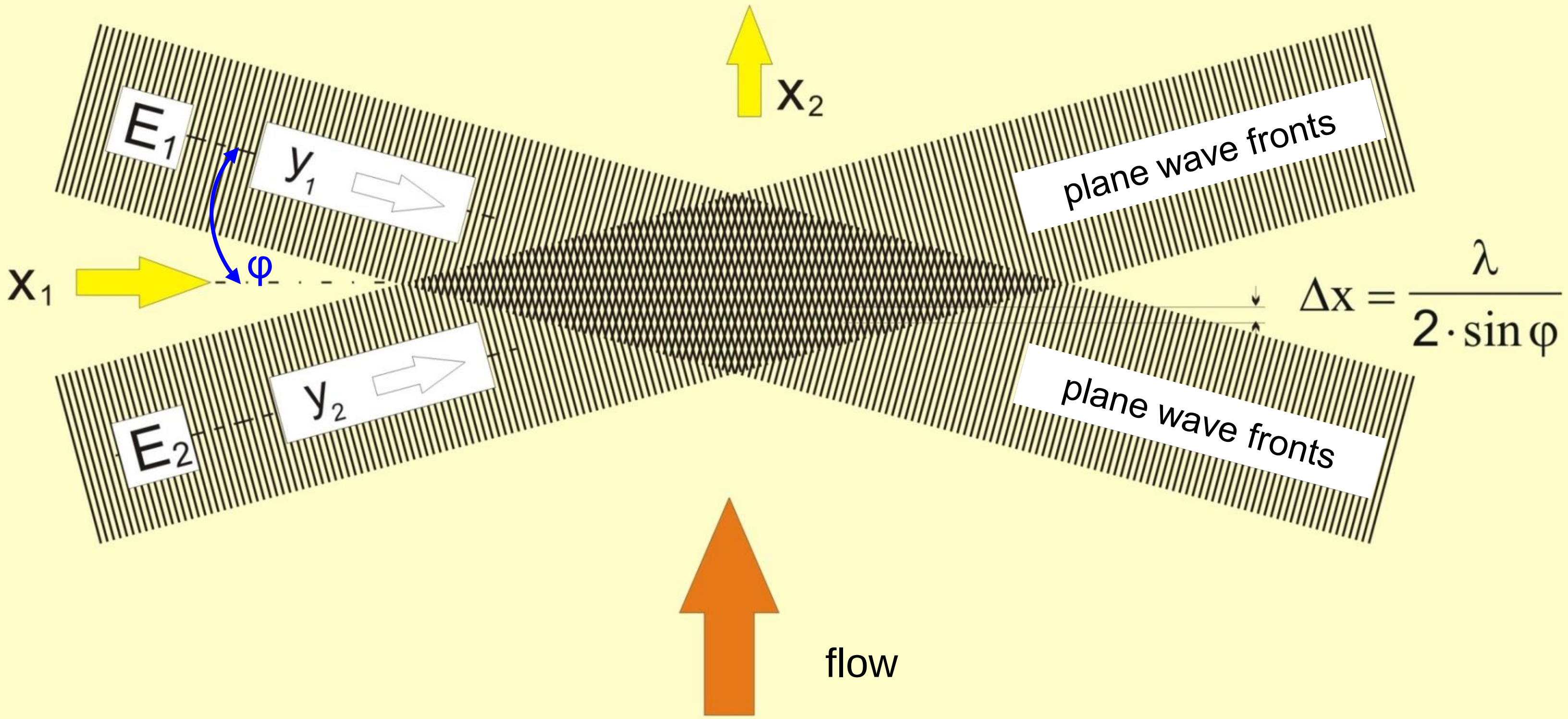
for $f_1 \approx f_2$

$$E_{1/2} = 2 \cdot E_0 \cdot \cos\ 2\pi \left(f_1 \cdot t - \frac{x}{\lambda_1} \right) \cdot$$

$$\cos\ 2\pi \left(\frac{f_D}{2} \cdot t - \frac{\lambda_D}{2 \cdot \lambda_1^2} \cdot x \right)$$

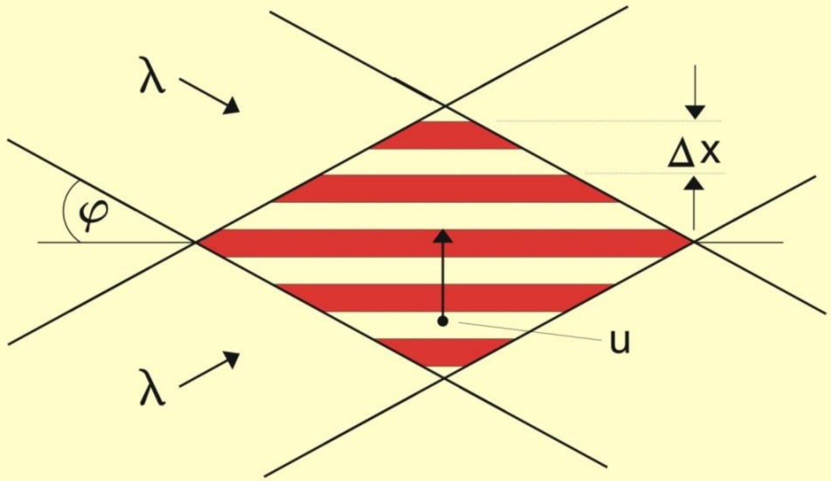
Explanatory Modell II: Interference Fringes Modell

(→ 'Engineer's explanation')



<u>interference fringes:</u> resulting pattern of constructive and destructive interference of coherent waves

Explanatory Modell II: Interference Fringes Modell



stationary interference fringes in the intersection of the two laser beams (measurement volume)

separation of interference fringes

$$\Delta x = \frac{\lambda}{2 \sin \varphi}$$

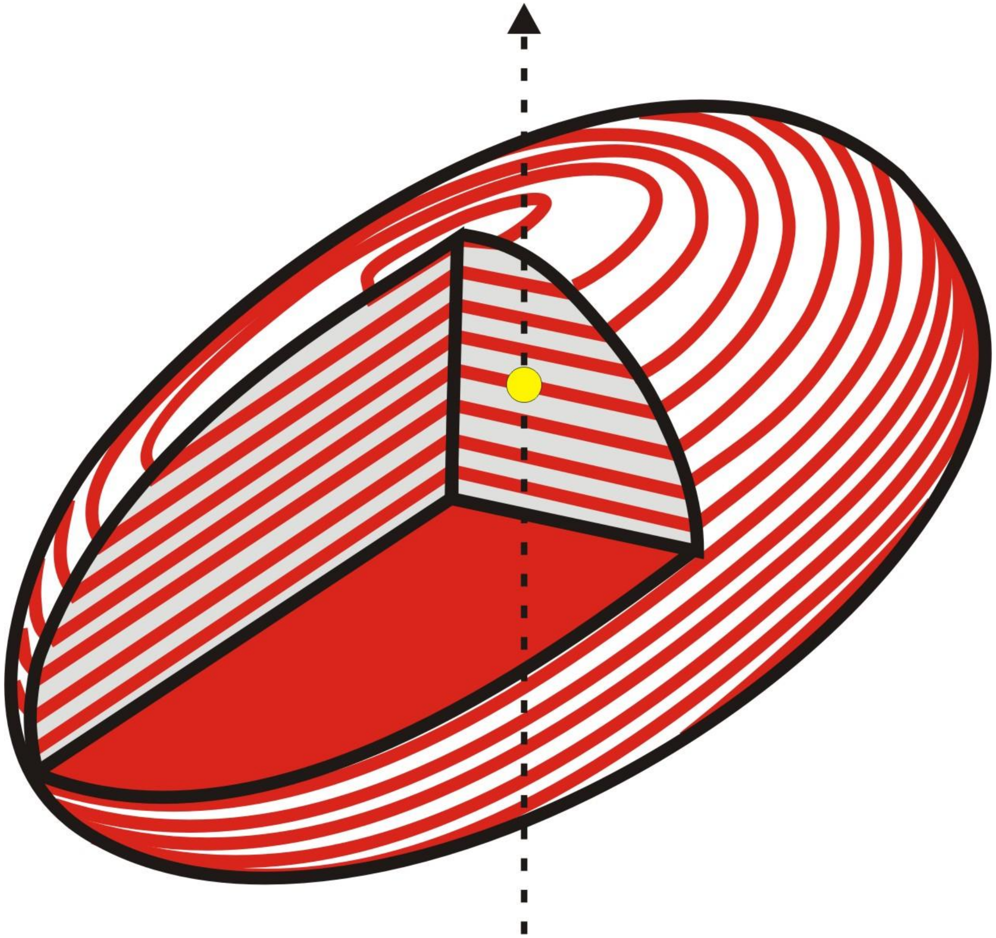
a particle moving with velocity u perpendicular through the interference fringes scatters the light of the bright fringes (constructive interference) with a frequency of

$$f_D = \frac{u}{\Delta x} = \frac{2\, u \sin \varphi}{\lambda}$$

$$u = \frac{f_D \, \lambda}{2 \sin \varphi} = k \, f_D$$

Intensity Distribution in Measurement Volume

The measurement volume is a spheroid (rotational ellipsoid), i.e. a body with ellipsoid surface.

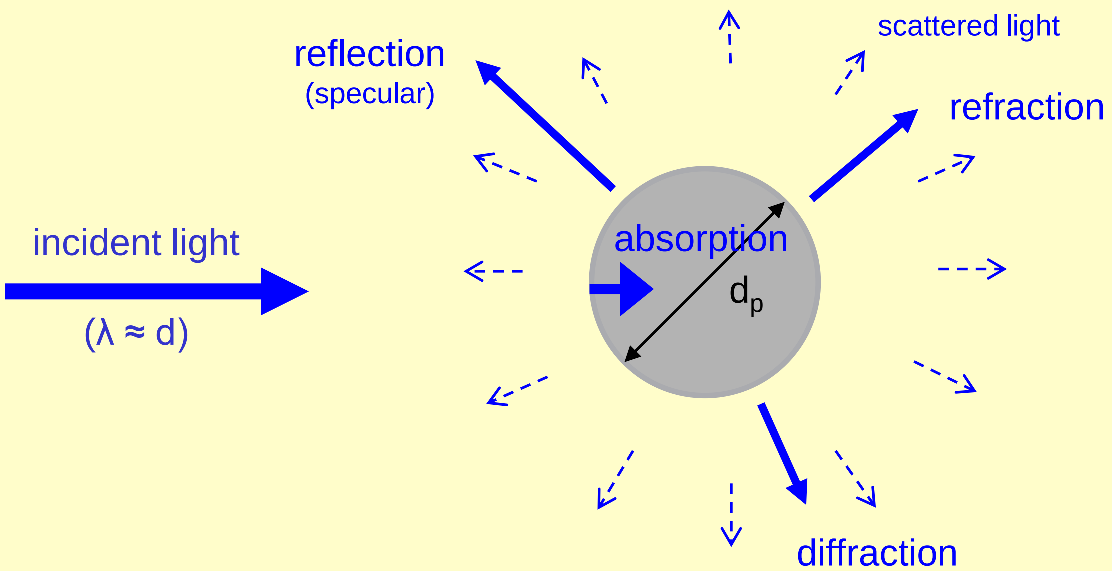


https://commons.wikimedia.org/wiki/File:
Palla_da_Rugby.jpg

Interaction of Light and Matter – Scattering Characteristics

A main distinguishing feature for light-matter interaction is the ratio of object size to light wavelength.

For gas flows where seeding particles with diameters of few micrometer (~ 0.5 – 3.0 µm) are employed, the object size and the visible light wavelength are of the same order of magnitude. The light-matter interaction is then dominated by scattering, or more precisely by Mie scattering (or Lorenz-Mie scattering). In terms of light, scattering means a process which results in a deflection from a straight trajectory.



light-matter interactions:

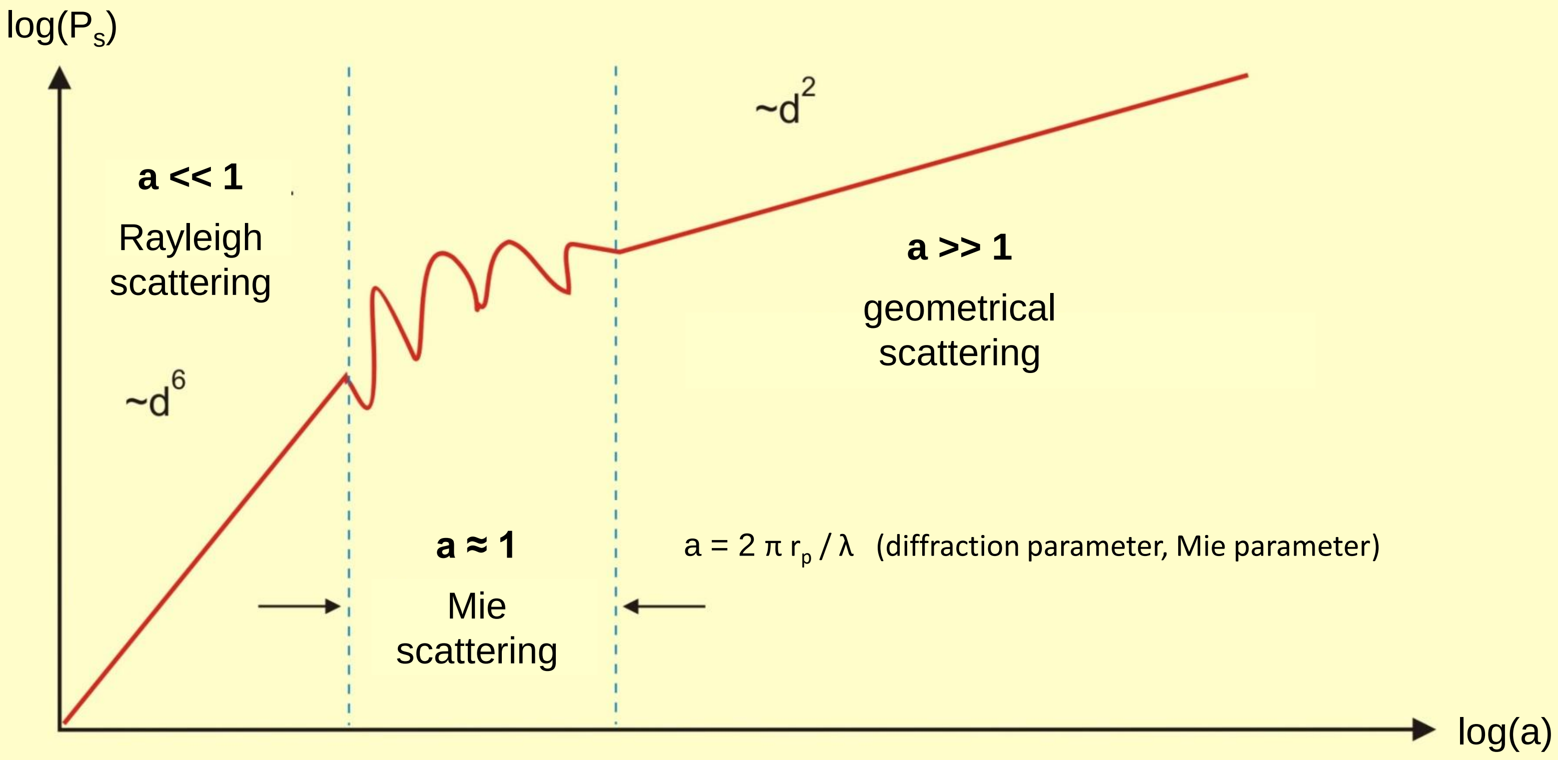
- scattering
- directed reflection (specular) (GO)
- diffuse reflection
- refraction (GO)
- diffraction (PO)
- absorption

GO: geometrical optics (light rays)
PO: physical optics (wave nature of light)

dimensionless size parameter: $a = 2\,\pi\, r_p / \lambda = \pi\, d_p / \lambda$ (diffraction parameter, Mie parameter)

- r_p particle radius
- λ wave length of light

Power of scattered Light and Particle Size

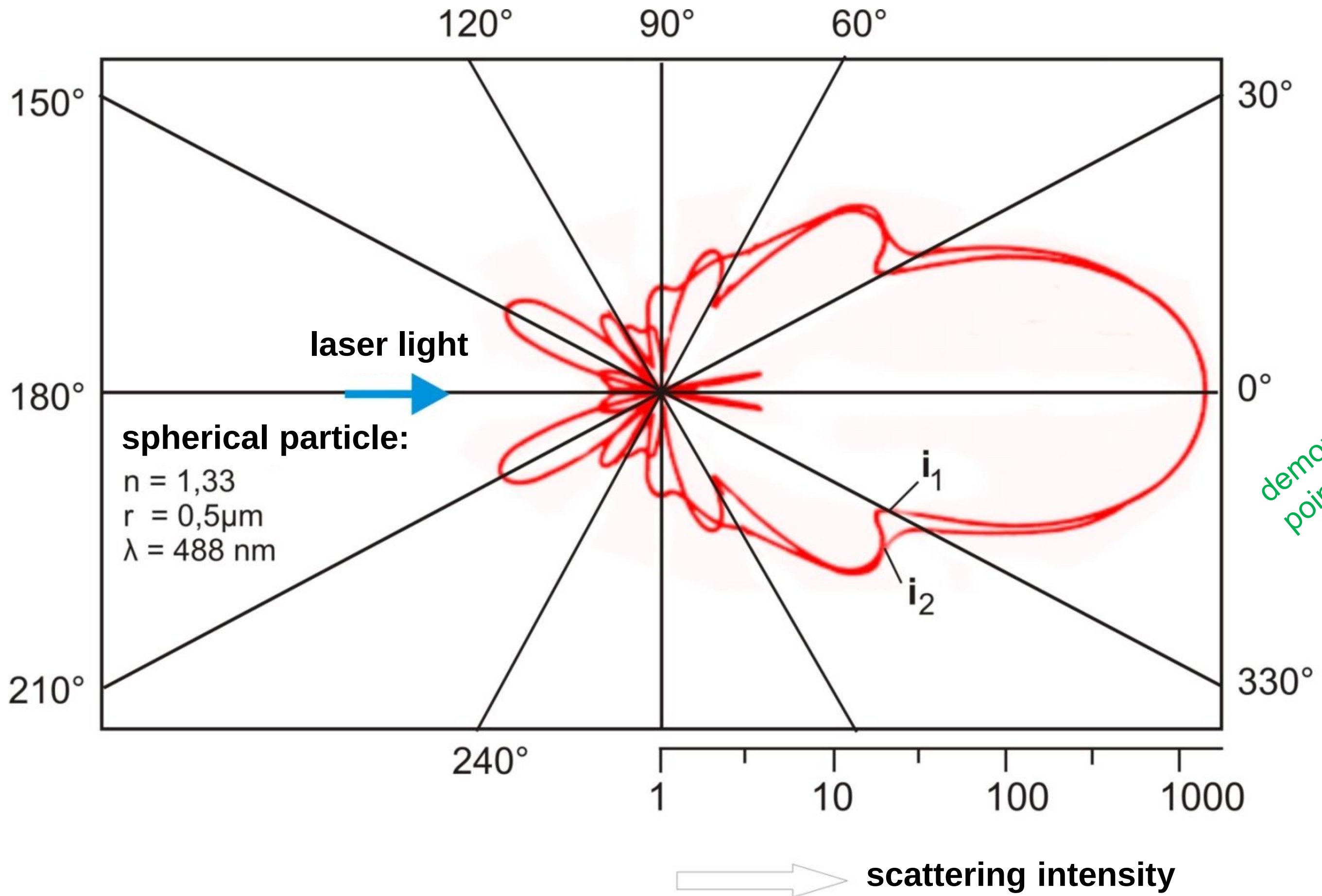


range of *Mie scattering* (Gustav Mie 1906) depends on

- optical material / substance properties
- direction, polarization
- for water droplet (wd) and visible light: $0.5 < d_{wd} < 3.0\ \mu m$

Directional Distribution of Scattering Intensity

for water droplet of d_{wd} = 1 µm according to Mie theory



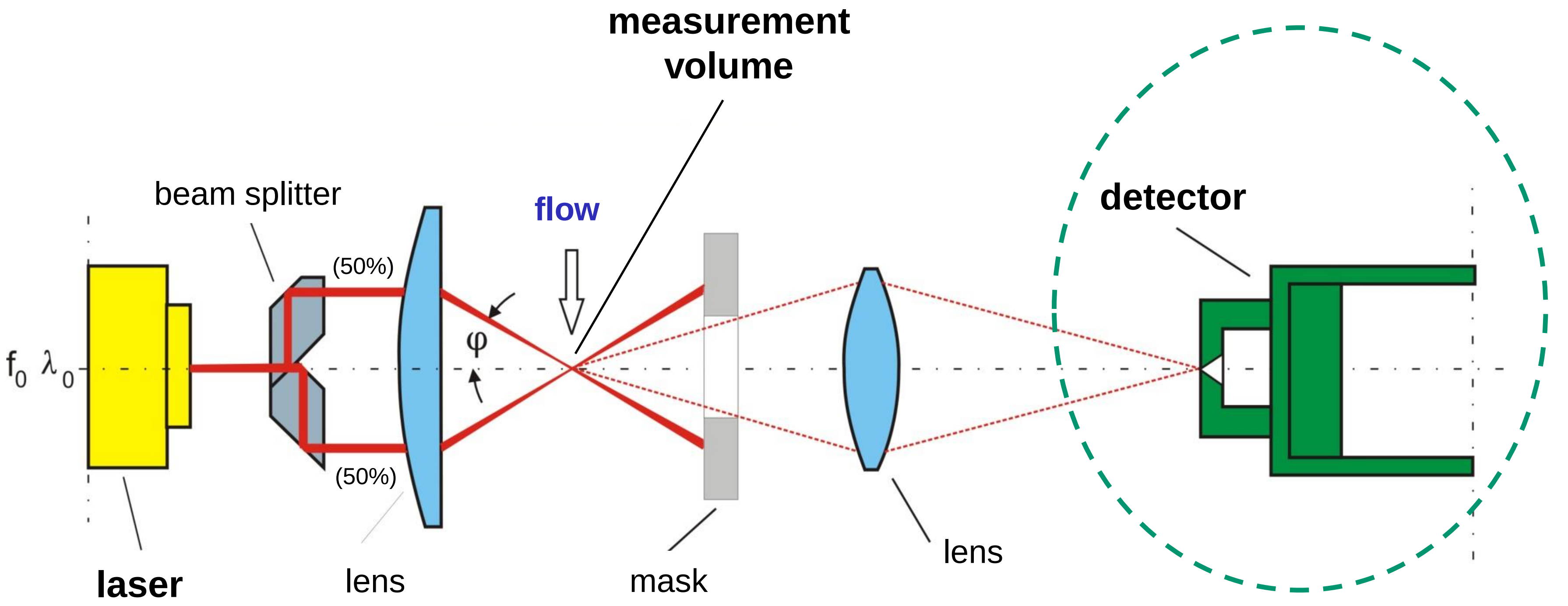
demonstrate with laser pointer and chalk dust

i_1 and i_2: scattering intensity perpendicular and parallel to polarization plane of incident light

Directional Distribution of Scattering Intensity

Signal Detection

Schematic setup of LDV system

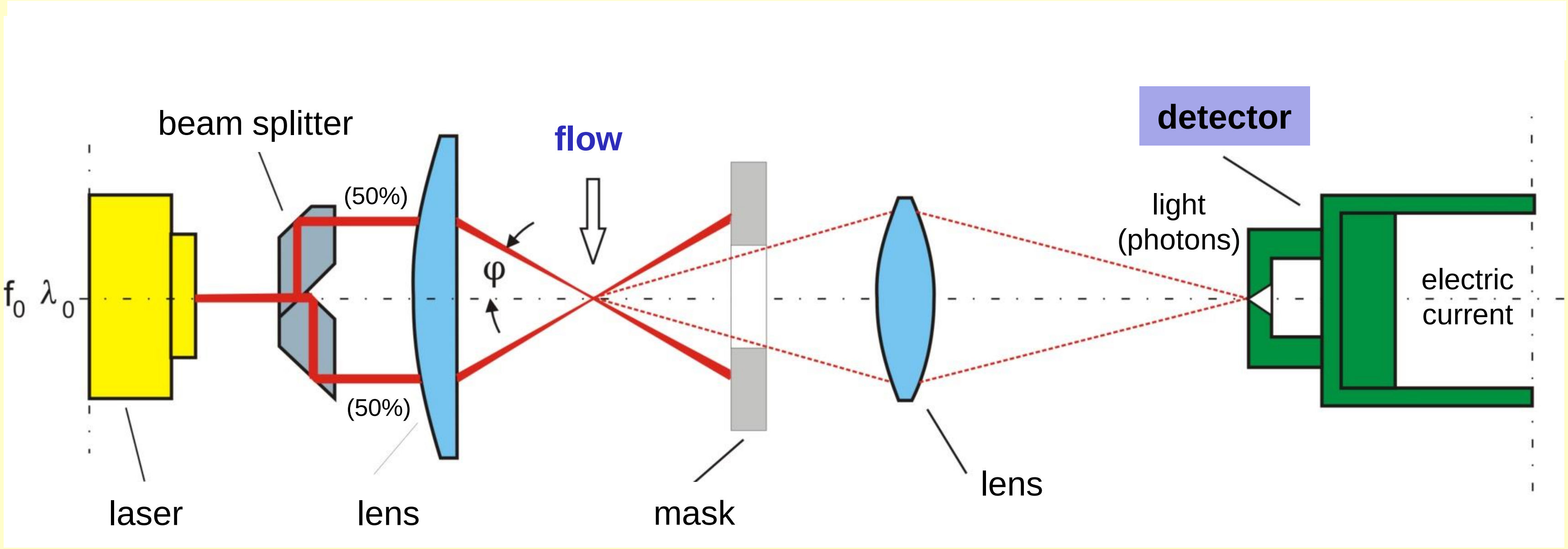


Conversion of Light (Photons) into Electrical Current

Incident photons are converted into a electrical signal. Theoretical basis is the Photoelectric Effect which describes Light-Matter interactions.

In LDV systems two types of detectors are employed

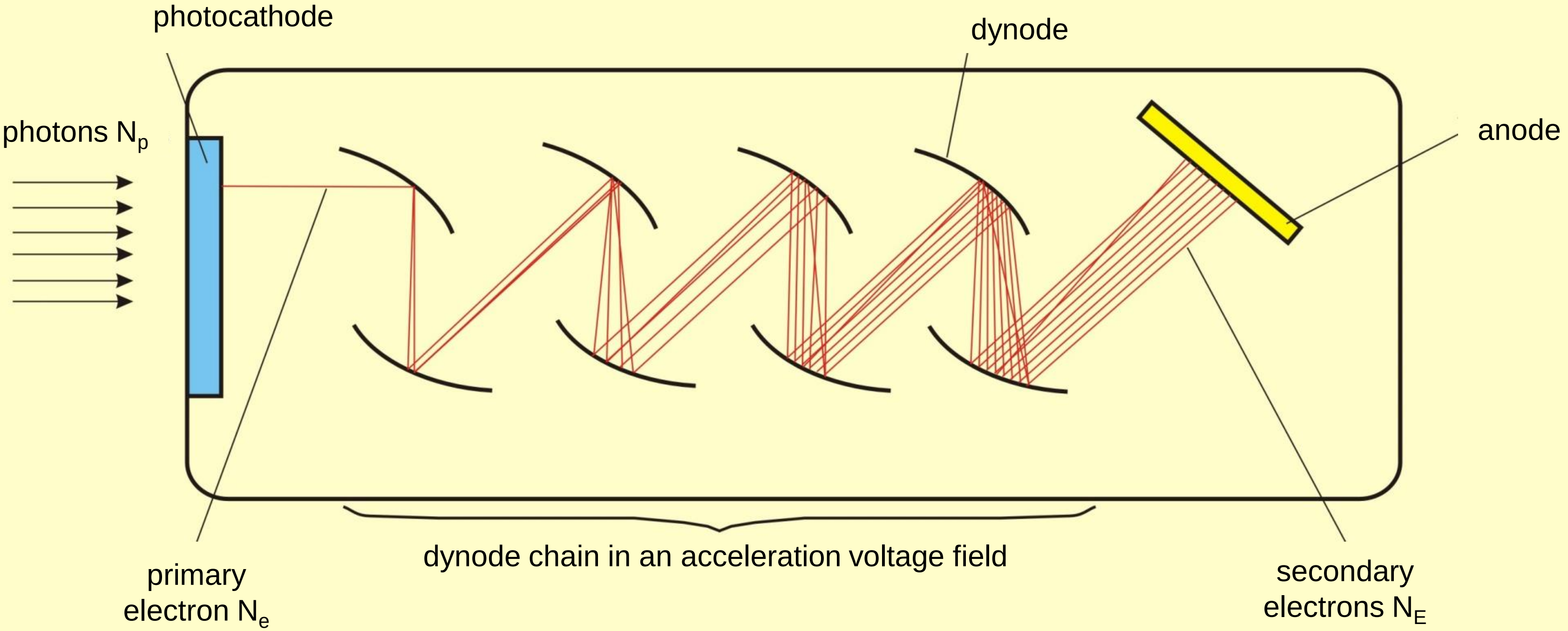
- photomultiplier (external photoelectric effect)
- photodiode (internal photoelectric effect)



Photomultiplier Tube (PMT)

External photoelectric effect:

Ejection of electrons from the surface of a solid by the absorption of a sufficient amount of photons.

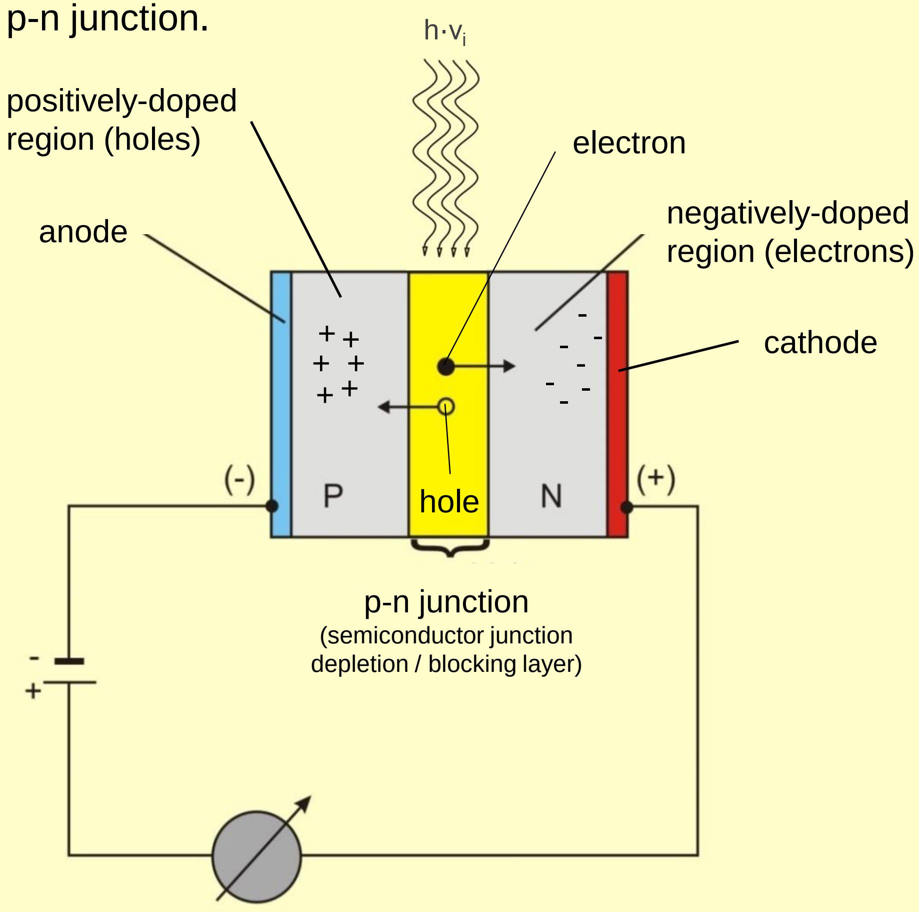


rate of electrons	$N_e = \eta_Q N_p$	η_Q quantum yield
rate of secondary electrons	$N_E = \eta_Q N_p \eta_Q^m$	m number of dynodes

Photodiodes

Internal photoelectric effect:

Photons are absorbed and excite electrons which move from the valence band to the conduction band (intrinsic photoelectric effect), from the valence band to impurity levels or from the impurity levels to the conduction band (extrinsic photoelectric effect) - i.e. photons are not ejected but remain in the 'material'. A photodiode is a special (light sensitive) semiconductor diode with a reversed-biased p-n junction.

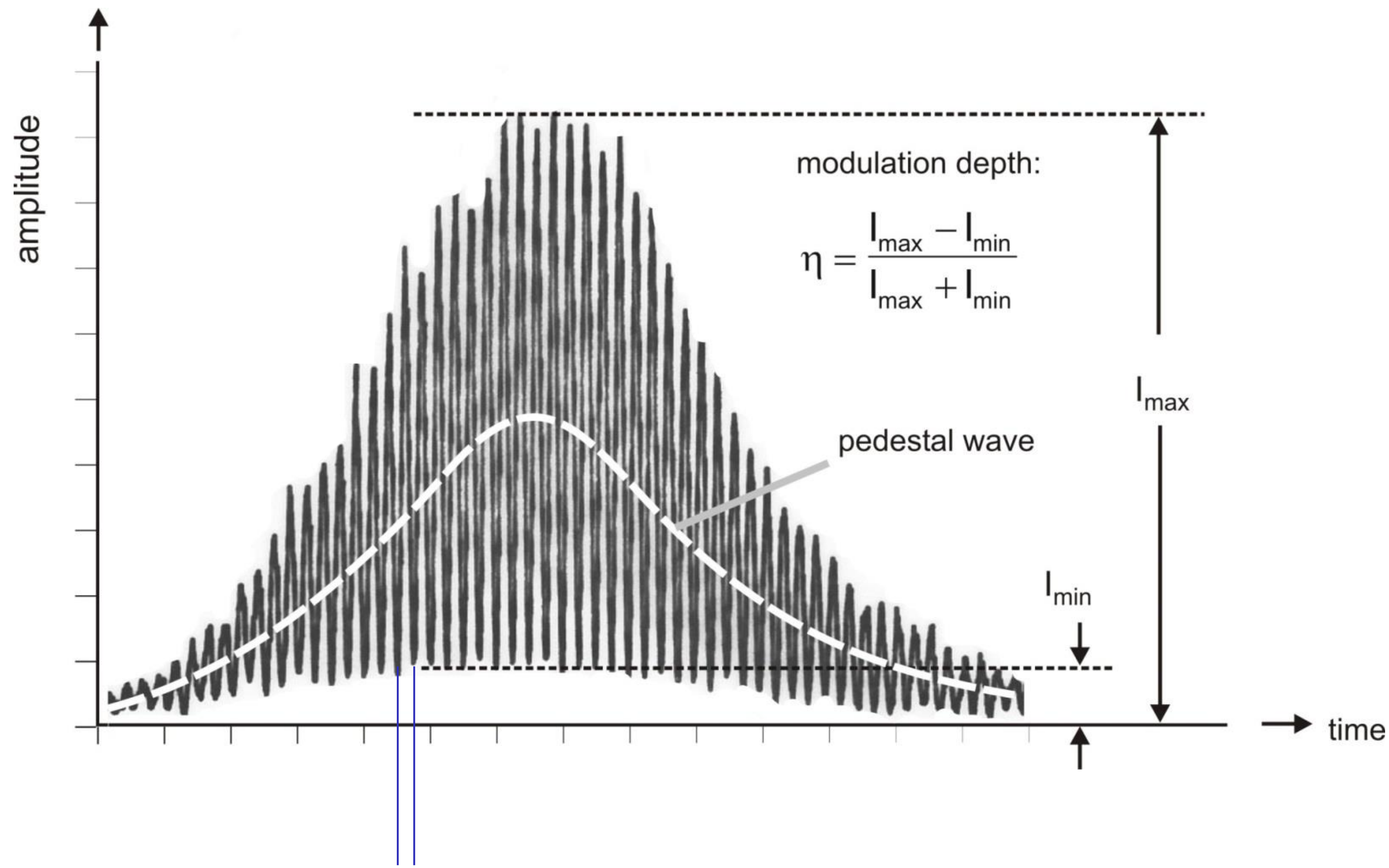


An incident photon with sufficient energy striking the depletion layer creates an *electron-hole pair* (internal photoelectric effect).

The free electron moves towards the cathode (*reverse-biased p-n junction*) and a electric current flows over the outer circuit. The electric current is linearly proportional to the radiant flux density of the incident light.

Laser Doppler Velocimetry Signal

Laser Doppler Velocimetry signal is referred to as **burst**.



$\Delta t = 1/f_D$ f_D: LDV signal frequency (‘Doppler frequency’)

Special Aspects

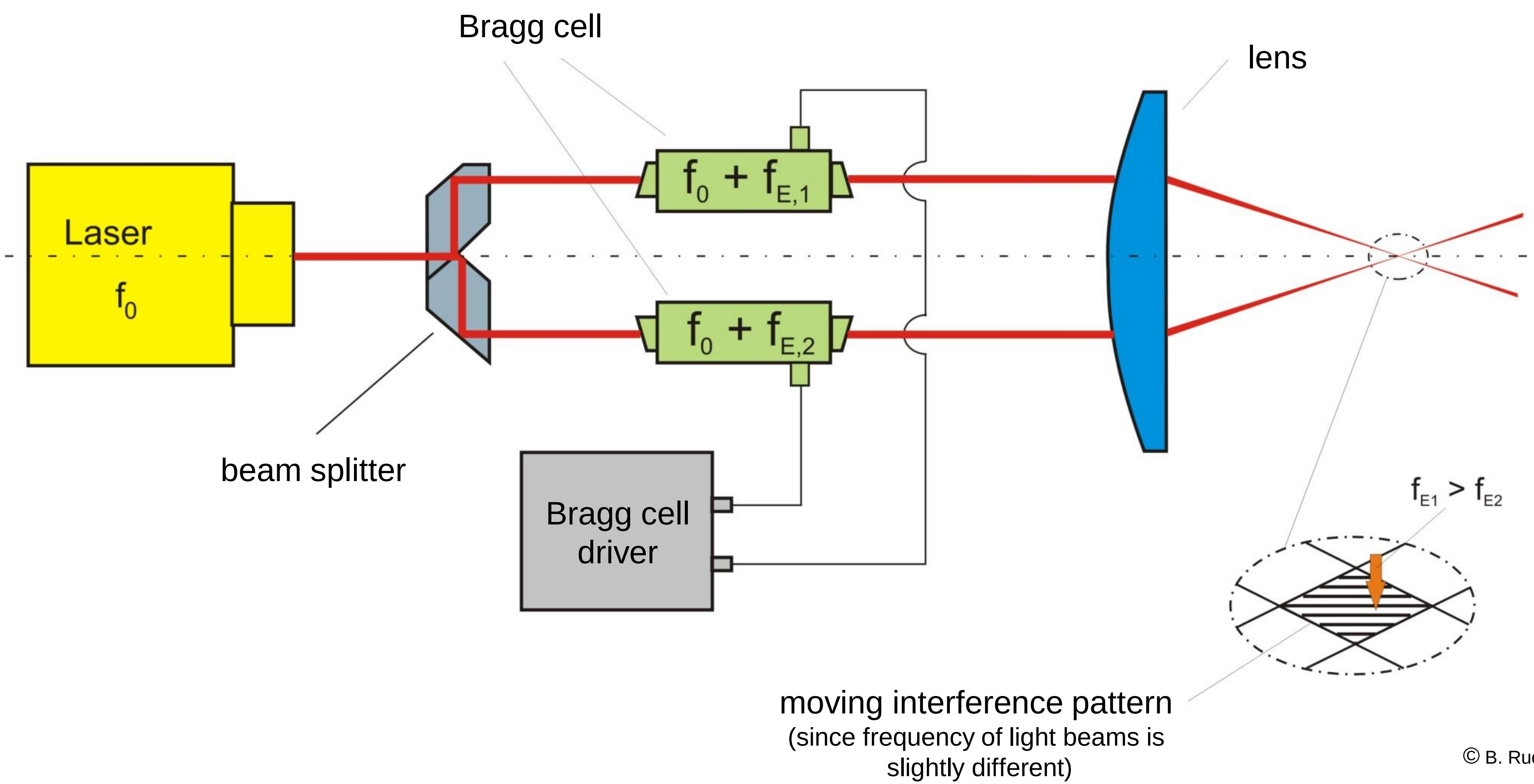
Discrimination of Flow Orientation

Discrimination of Flow Orientation

Need to distinguish the flow orientation (positive / negative) in turbulent flows or in flow reversals.

Frequency shifting with Bragg cell

(acousto-optic modulator - AOM)



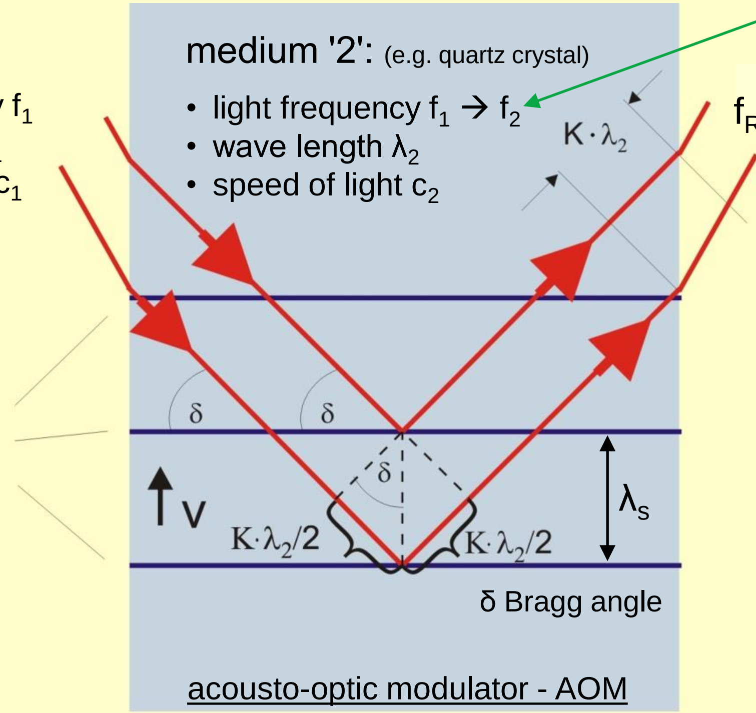
Operating Principle of Bragg Cell

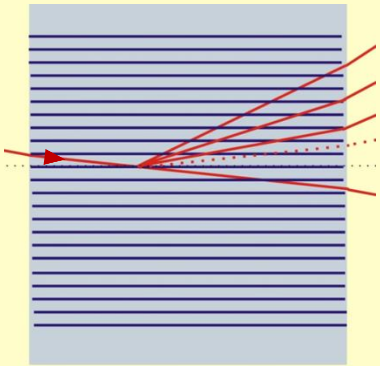
Light beam enters under an inclined incident angle a transparent medium with density differences and is reflected at density differences.

usually: frequency is invariant!
i.e. $f_2 = f_1$

medium '1' :

- light frequency f_1
- wave length λ_1
- speed of light c_1





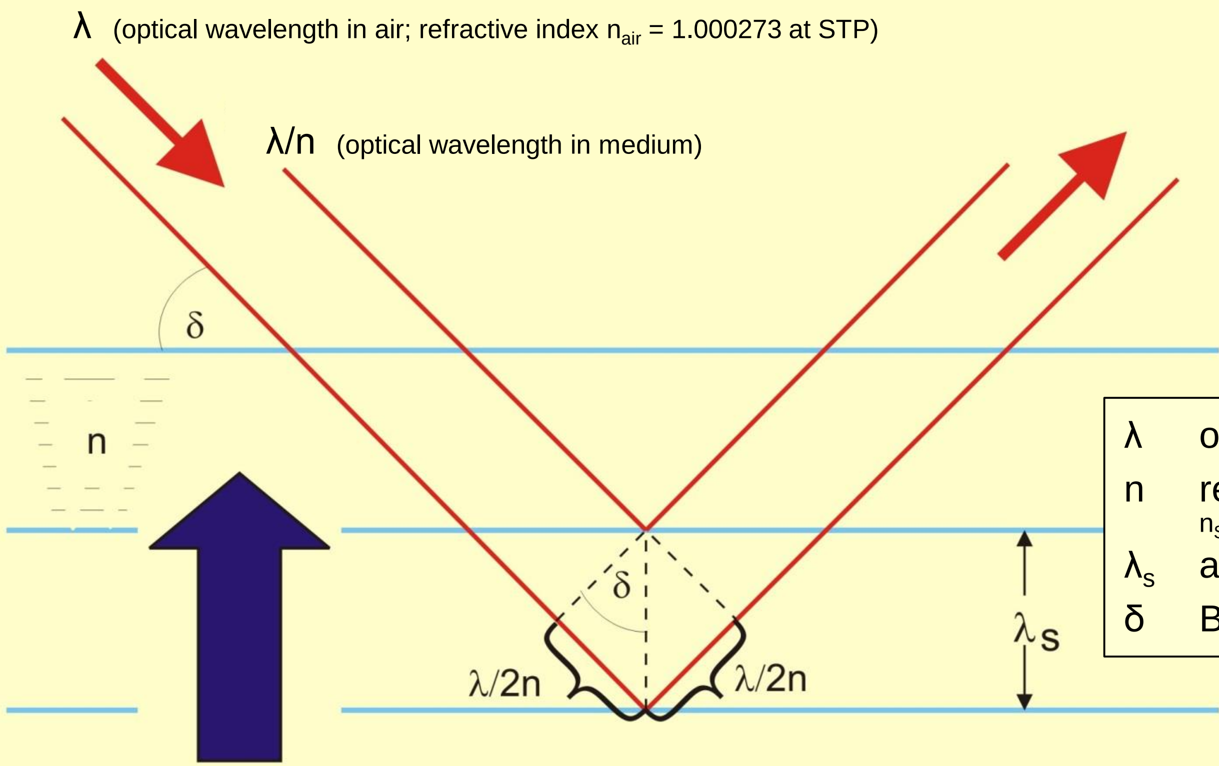
Bragg condition:

$$2 \cdot \lambda_S \cdot \sin\delta = \lambda_2$$

The incident light waves of a light beam are reflected at density differences. If the Bragg condition is fulfilled, reflected light waves leave with an integer difference in wavelength and constructively interfere such that an emergent beam appears. Several emergent light beams appear at selected exit angles (K-th order).

Operating Principle of Bragg Cell

Bragg cell: Reflection at acoustic wave fronts (density differences)



λ	optical wavelength in air
n	refractive index $n_{SiO2} \approx 1.45$ [n = n(f)] (SiO_2: quartz)
λ_s	acoustic wavelength
δ	Bragg angle

acoustic wave

- wave length λ_s
- excitation frequency f_s

→ causes density differences

→ causes change of refractive index n

Bragg condition: $$2 \cdot \lambda_s \cdot \sin\delta = \frac{\lambda}{n}$$

approximation in medium: $$\delta = \frac{\lambda}{2 \cdot \lambda_s \cdot n}$$
(δ small)

Operating Principle of Bragg Cell

The density differences are not stationary (in space) but travel with the acoustic wave, i.e. the density differences occur with a certain frequency at a fixed location in the crystal (medium '2'). This affects the light frequency in the emergent beam orders.

Doppler effect occurring at density front

incident on front:

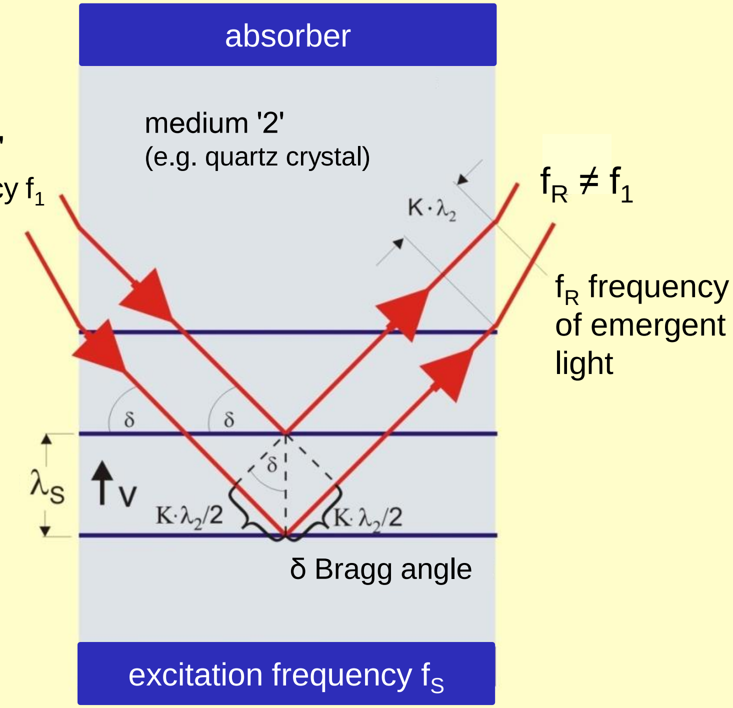
$$f' = f \cdot \left(1 + \frac{v \cdot \sin\delta}{c_2}\right)$$

outgoing from front:

$$f_R = \frac{f \cdot \left(1 + \frac{v \cdot \sin\delta}{c_2}\right)}{\left(1 - \frac{v \cdot \sin\delta}{c_2}\right)}$$

v: velocity of density fluctuations in medium '2'

c_2: speed of light in medium '2'



Taylor series expansion:

$$f_R \approx f \cdot \left(1 + \frac{2 \cdot v \cdot \sin\delta}{c_2}\right) = f + \frac{f}{c_2} \cdot 2 \cdot v \cdot \sin\delta$$

with $\sin\delta = \frac{K \cdot \lambda_2}{2 \cdot \lambda_S} = \frac{K \cdot \lambda_1 \cdot n_1}{2 \cdot n_2 \cdot \lambda_S}$ (K: order of beam)

$$\rightarrow \quad f_R = f + \frac{f}{c_2} \cdot \frac{\lambda_2 \cdot v \cdot K}{\lambda_S} = f + K \cdot f_S$$

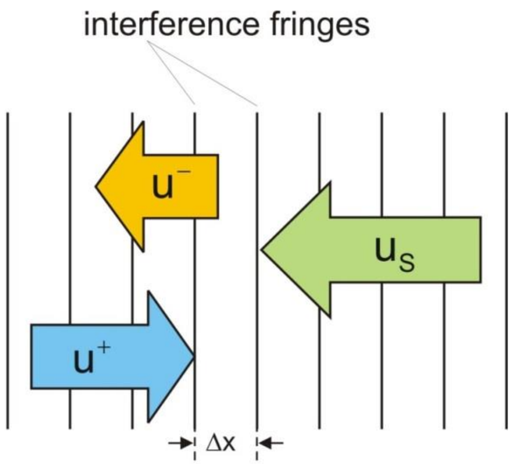
(f_S: excitation frequency)

The Doppler equation is applied to the travelling wave front (density front).

The frequency of the emergent beam of order K is shifted by $K \cdot f_S$.

Selection of Frequency Shift of Bragg Cell

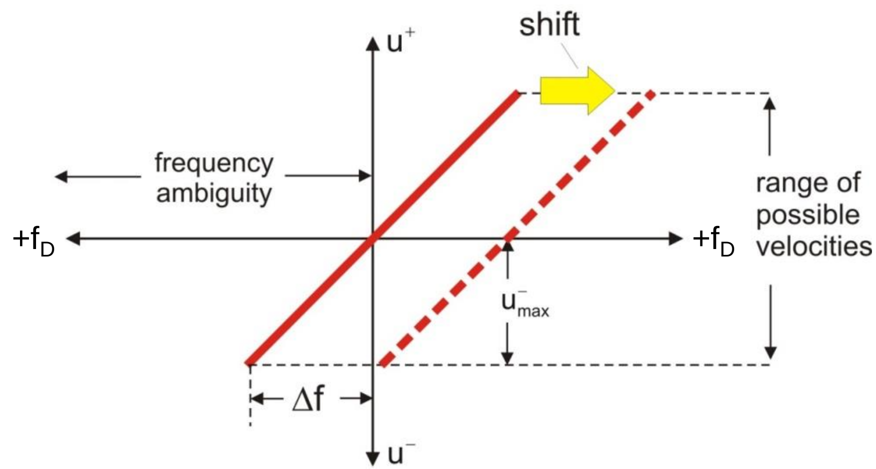
LDV signal frequency ('Doppler' frequency) $f_D = \frac{u}{\Delta x} = \frac{2\,u \sin\varphi}{\lambda}$



u_S : velocity of fringes in measuring volume

u^+ : maximum velocity of flow in forward direction

u^- : maximum velocity of flow in backward direction



$$\Delta f > \frac{2\,|\bar{U}_{max}|\ \sin(\varphi)}{\lambda}$$

→ frequency shift $f_S \geq \Delta f$

Particle Following Behavior

(relevant for LDV, PIV, PTV)

Characteristics of Particle Following Behavior

Why is particle following behavior relevant? Because seeding particles should follow the flow ideally!

Relaxation time ('ideal' definitions)

- Particle enters a fluid at rest where a resistance force is acting. After a certain time – the *relaxation time* – the particle comes at rest.
- Particle falls from rest position into a fluid. After a certain time – the *relaxation time* – the particle reaches a constant speed of fall.

Stopping distance (corresponds to 1st definition of relaxation time)

- Particles enters a fluid at rest where a resistance force is acting. The path length that the particle travels until coming to 'rest' is referred to as *stopping distance*.

Stokes number

- The particle following behavior is assessed relative to a characteristic length of the flow systems. For this purpose, the *stopping distance* is related to a characteristic length of the flow system.

Basset-Boussinesq-Oseen equation (BBO)

- The exact equation of motion for a particle in a fluid motion with harmonic fluctuations of different frequencies and amplitudes is established. The solution are frequency-dependent amplitude and phase ratios between particle and fluid motion characterizing the particle following behavior.

Stopping Distances and Stokes Number

The **stopping distance X_s** is the distance which a particle travels until it decelerates to 37% of its initial velocity u_0 or accelerates to 63% of its terminal velocity u_0. With the definitions of the *relaxation time(s)*, the stooping distance can be calculated according to

$$X_s = u_0 \cdot \tau$$

A meaningful assessment of the particle following behavior can be obtained by relating the stopping distance X_s to a characteristic length scale of the fluid system L_{char}. This dimensionless number is called **Stokes number STK**.

$$STK = X_s / L_{char} = u_0 \cdot \tau / L_{char}$$

The Stokes number combines the particle inertia and the fluid viscosity with a characteristic length scale of the fluid system. Small Stokes numbers STK << 1 indicate a 'good' particle following behavior.

Basset-Boussinesq-Oseen Equation (BBO)

The BBO equation describes the motion of a particle in an unsteady flow (at small Re-numbers).

$$\overbrace{\frac{\pi}{6}\cdot d_p^3\cdot\rho_p\cdot\frac{du_p}{dt}}^{\text{inertia force}} = \overbrace{3\cdot\pi\cdot\mu_f\cdot d_p\cdot(u_f-u_p)}^{\text{Stokes drag}} + \overbrace{\frac{\pi}{6}\cdot d_p^3\cdot\rho_f\cdot\frac{du_f}{dt}}^{\substack{\text{pressure gradient force}\\ \text{due to acceleration of fluid}}} + \overbrace{\frac{\pi}{12}\cdot d_p^3\cdot\rho_f\cdot\left(\frac{du_f}{dt}-\frac{du_p}{dt}\right)}^{\substack{\text{added mass force}\\ \text{due to resistance of surrounding}\\ \text{fluid against acceleration}}} +$$

$$+\underbrace{\frac{3}{2}\cdot d_p^2\cdot(\pi\cdot\rho_f\cdot\mu_f)^{1/2}\cdot\int_{t_0}^{t}\frac{\left(\frac{du_f}{dT}-\frac{du_p}{dT}\right)}{(t-T)^{1/2}}\cdot dT}_{\substack{\text{Basset history term}\\ \text{due to perpetual transformation / reconfiguration}\\ \text{of the boundary layer around the particle}}}$$

d_p	particle diameter
ρ_p	particle density
ρ_f	fluid density
u_p	particle velocity
u_f	fluid velocity
μ_f	dynamic viscosity of fluid

Maxey MR, Riley JJ (1983) Equation of motion for a small rigid sphere in a nonuniform flow. Physics of Fluids Vol. 26, pp. 883-889.

<u>remarks:</u>

1.) for steady motion: only Stokes drag is acting on particle

2.) pressure gradient force, added mass force, Basset history term scale with fluid density

→ are often neglected for liquid or solid particles in gases

Basset-Boussinesq-Oseen Equation (BBO)

The solution of the BBO equation for a particle in a fluid motion with harmonic fluctuations according to Hjelmfelt and Mockros (1966) yields expressions for the amplitude ratio and phase angle. The amplitude ratio relates the amplitude of the fluctuation of the particle motion to the fluctuation in the fluid. The phase angle indicates the phase shift between particle and fluid motion.

amplitude ratio η = η(ω)

$$\eta = \sqrt{\left(1 + f_1\right)^2 + f_2^2}$$

phase angle β = β(ω)

$$\beta = \tan^{-1}\left(\frac{f_2}{1 + f_1}\right)$$

N_{St} Stokes number (not stopping distance STK! see below)

ρ_p particle density

ρ_f fluid density

u_p particle velocity

u_f fluid velocity

ν_f kinematic viscosity of fluid

ω 2 π n, with n fluctuation frequency

$$f_1 = \frac{\left[1 + \dfrac{9}{\sqrt{2}\cdot\left(s + \dfrac{1}{2}\right)}\cdot N_{St}\right]\cdot\left[\dfrac{1-s}{s + \dfrac{1}{2}}\right]}{\dfrac{81}{\left(s + \dfrac{1}{2}\right)^2}\cdot\left[2\cdot N_{St}^2 + \dfrac{N_{St}}{\sqrt{2}}\right]^2 + \left[1 + \dfrac{9}{\sqrt{2}\cdot\left(s + \dfrac{1}{2}\right)}\cdot N_{St}\right]^2}$$

$$f_2 = \frac{\dfrac{9\cdot(1-s)}{\left(s + \dfrac{1}{2}\right)^2}\cdot\left[2\cdot N_{St}^2 + \dfrac{N_{St}}{\sqrt{2}}\right]}{\dfrac{81}{\left(s + \dfrac{1}{2}\right)^2}\cdot\left[2\cdot N_{St}^2 + \dfrac{N_{St}}{\sqrt{2}}\right]^2 + \left[1 + \dfrac{9}{\sqrt{2}\cdot\left(s + \dfrac{1}{2}\right)}\cdot N_{St}\right]^2}$$

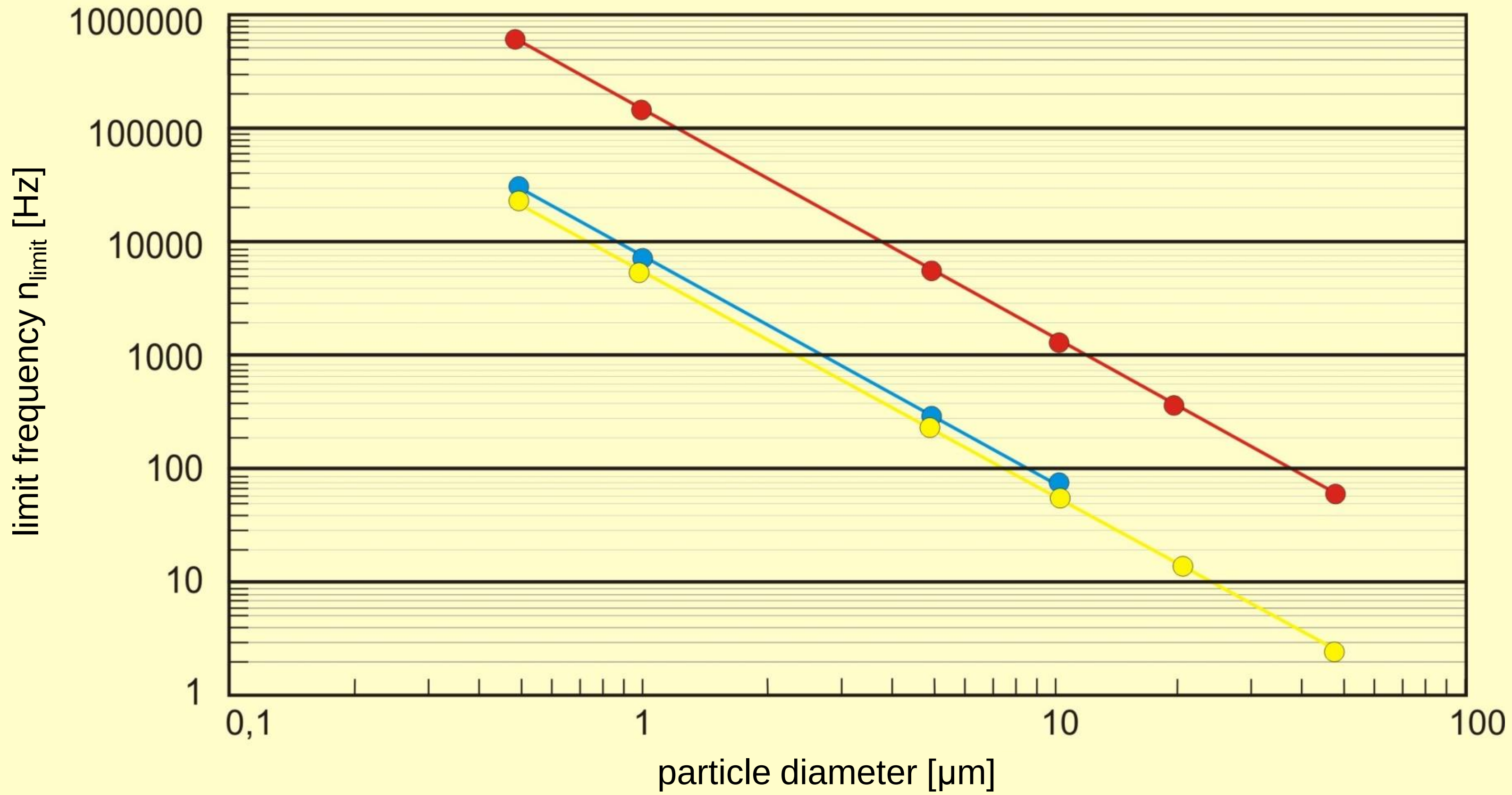
density ratio $s = \dfrac{\rho_p}{\rho_f}$

Stokes number $N_{St} = \sqrt{\dfrac{\nu_f}{\omega\cdot d_p^2}}$

Hjelmfelt AT, Mockros LF (1966) Motion of discrete particles in a turbulent fluid. Applied Scientific Research Vol 16, pp. 149–161.

Solution of BBO Equation

Limit frequency n_{limit} up to which particle can follow the fluctuating fluid motion for a required / specified amplitude ratio η = 0.99



sand in water
s = 2.65

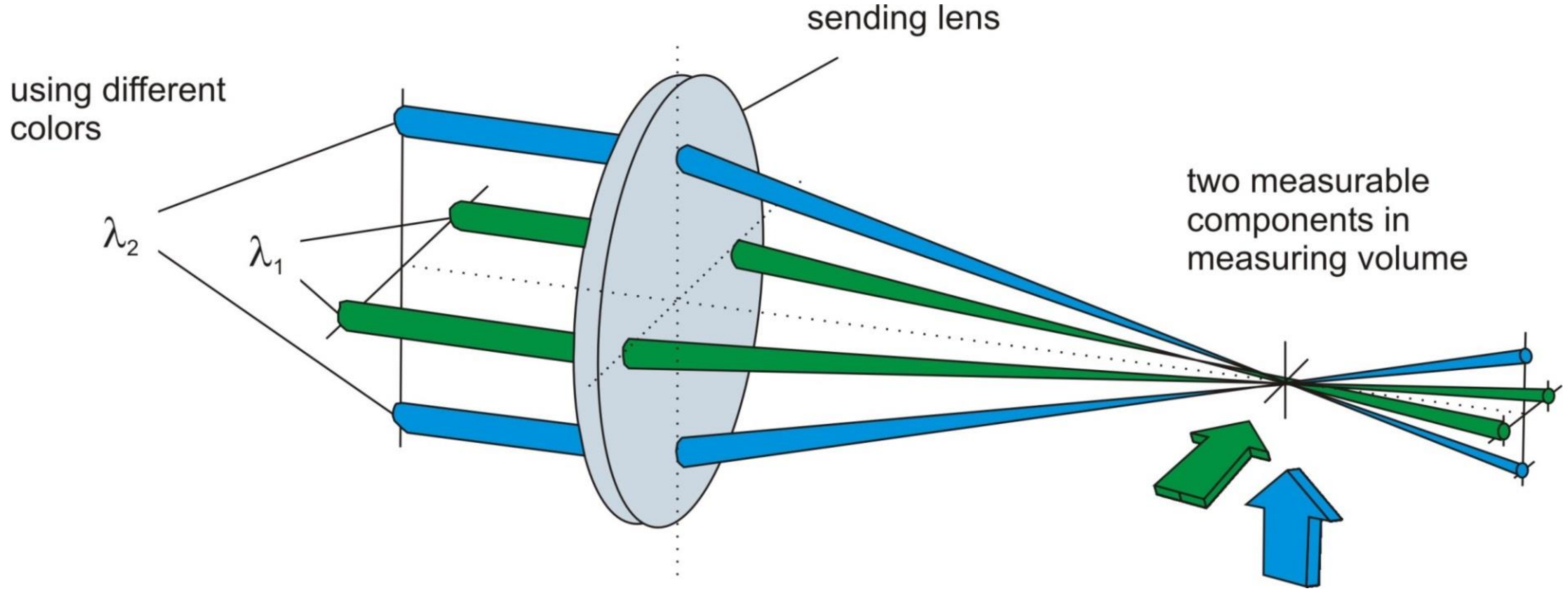
water in air
s ≈ 800

oil in air
s ≈ 700

Multi Component LDV Systems

2-Component (2D) LDV System

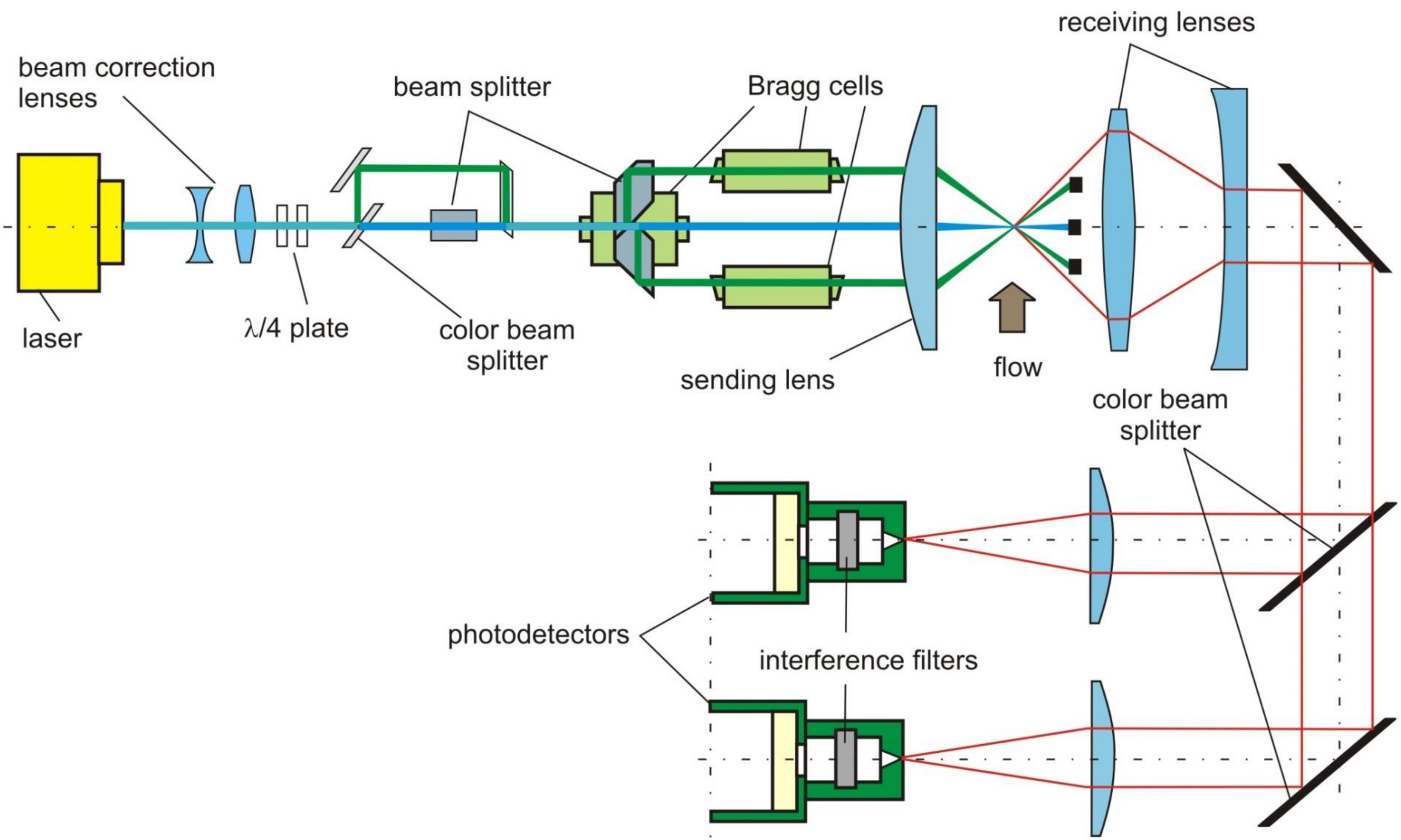
Two color two component LDV system



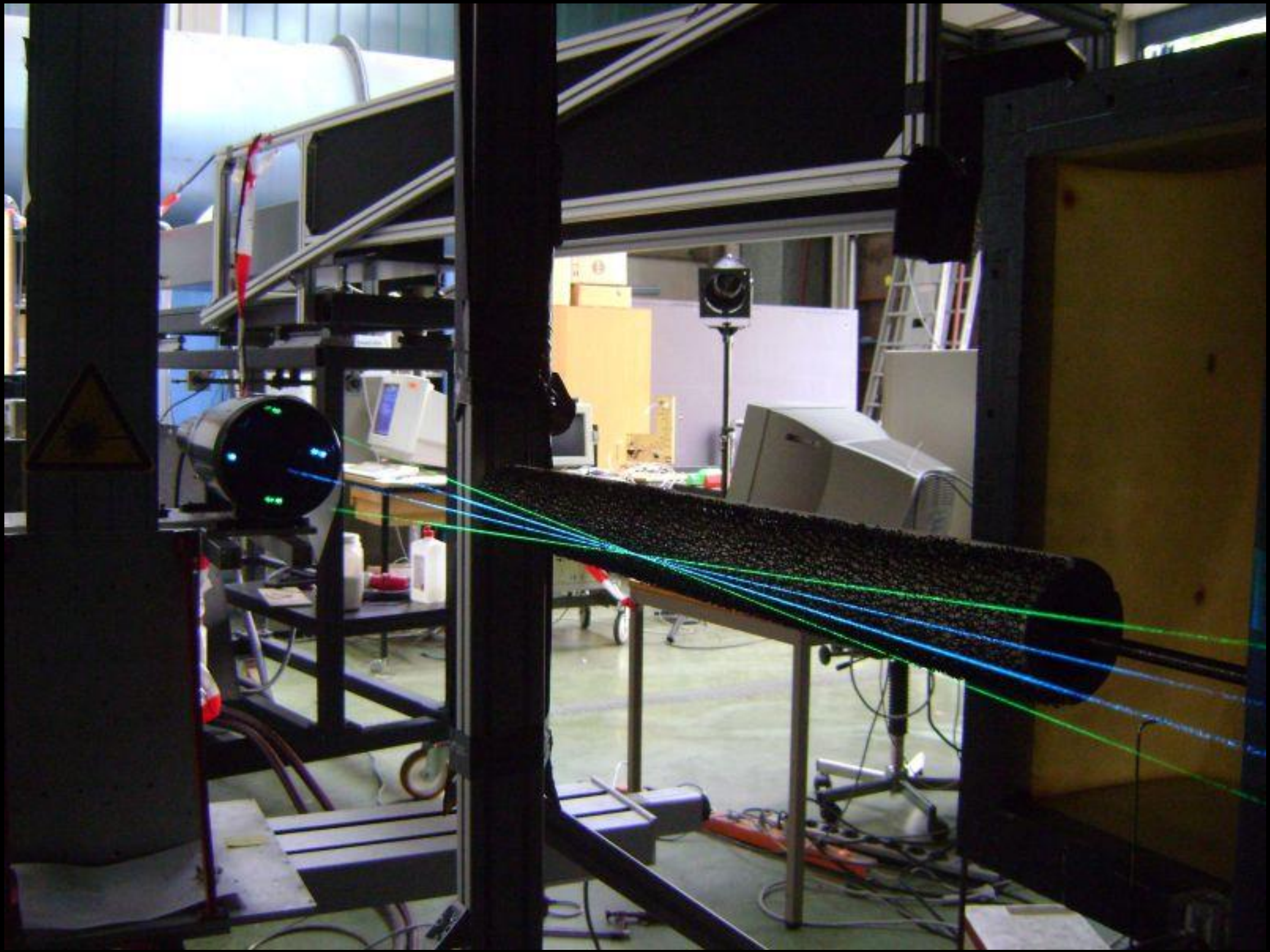
typical laser: Ar^+ Argon-ion gas laser

2-Component (2D) LDV System

Two color two component LDV system



2-Component (2D) LDV System



picture: Laboratory of Building and Environmental Aerodynamics, Institute for Hydromechanics, KIT

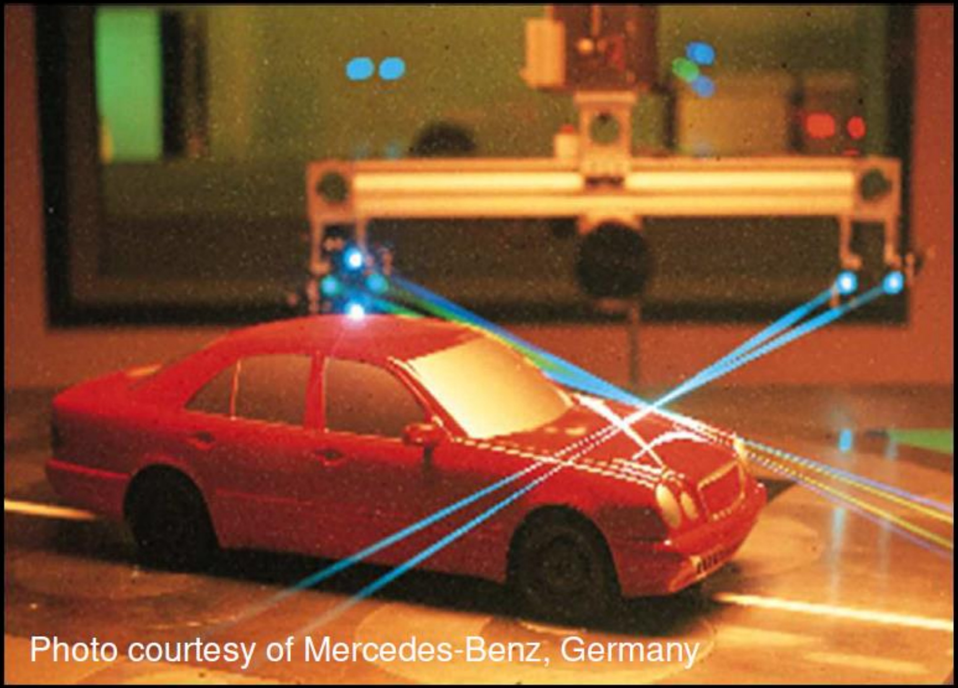
2-Component (2D) LDV System

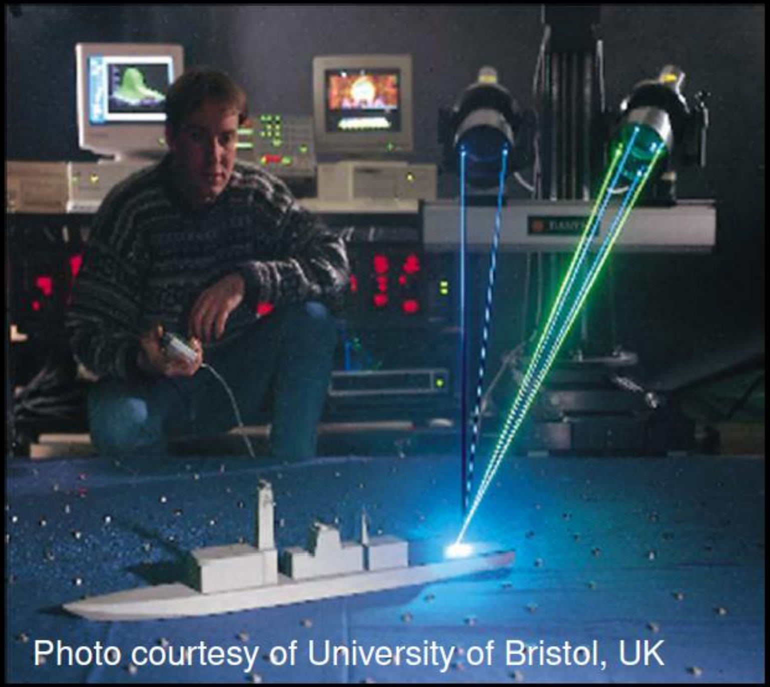
pictures: Laboratory of Building and Environmental Aerodynamics, Institute for Hydromechanics, KIT

2-Component (2D) LDV System

picture: Laboratory of Building and Environmental Aerodynamics, Institute for Hydromechanics, KIT

3-Component (3D) LDV System





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